Edison High School

IB Math Analysis Summer Assignment

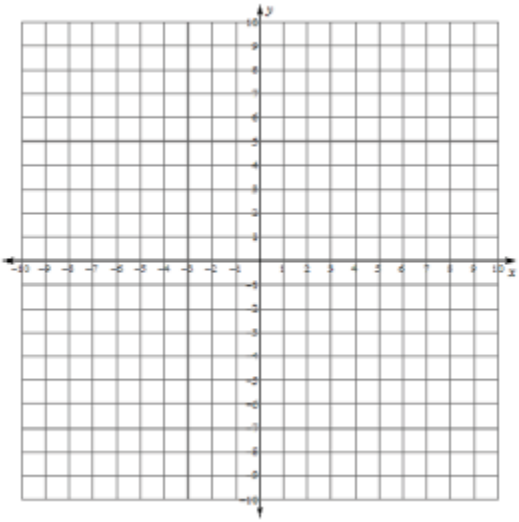
EDISON HIGH SCHOOL

EAGLES

Students entering IB Math Analysis 2 in the Fall of 2020 are encouraged to complete this packet for enrichment and course preparation. The material covered in this assignment is essential knowledge for success in IB Math Analysis, which is Part 2 of a 2 part course. If you have questions, you may contact Mrs. Racek at ktracek@fcps.edu.

FUNCTIONS

1. Given $f(x) = -|x+3| - 2$

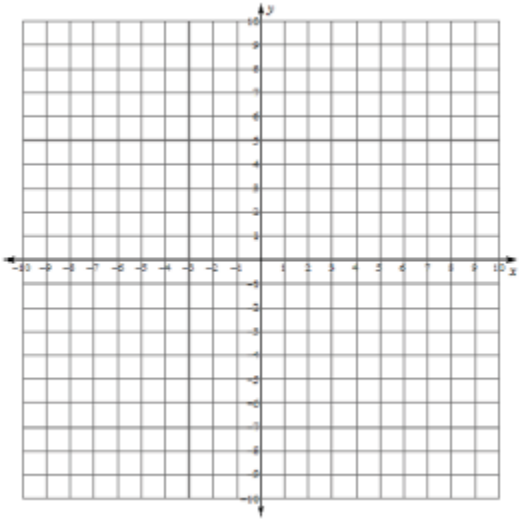


   a. Sketch
   b. Domain: _____
   c. Range: _____
   d. f(3) = _____
   e. f(x+5) = _____
   f. f(x) = -3, then x = _____

2.

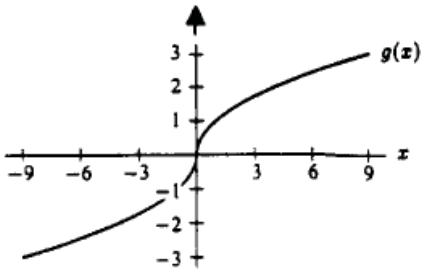
   a. Graph the piece-wise function

$$g(x) = \begin{cases} x^2 \ \ if \ x < 0 \\ \sqrt{x} \ \ if \ \ 0 < x < 4 \\ \frac{x}{2} \end{cases}$$



   b. g(-3) = _____
   c. g(1) = _____
   d. g(0) = _____
   e. Is g(x) a continuous function? _____

3. Given the graph of g(x) on the right

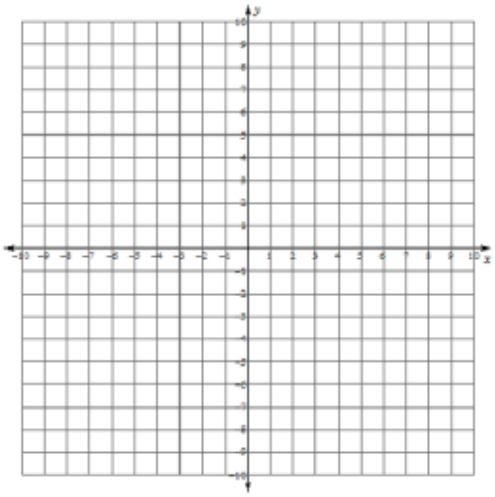


   a. Estimate $\frac{g(6)-g(0)}{6-0} =$

   b. The ratio in part (a) is the slope of a line segment joining two points on the graph. Sketch this line segment.

4. The rate at which water is entering a tank (t > 0) is represented by the given graph. A negative rate means that water is leaving the tank. State the interval(s) on which each of the following holds true:
    a. The volume of water is constant.
    b. The volume of water is decreasing.
    c. The volume of water is increasing.
    d. The volume of water is increasing fastest.

5. $Q(x) = \frac{3x}{x+1}$
    a. Where is this function discontinuous? _____
    b. State the equation if the vertical asymptote, x = _____
    c. State the equation if the horizontal asymptote, y = _____
    d. Sketch the graph.



    e. Write the equation of the inverse of Q(x) (Switch the x & y and then solve of y)

6. Use these functions: $f(x) - x + 1$ $g(x) = x^2\, 2x - 3$ $h(x) = 2x - 5$
    a. $f(3) =$
    b. $g(5) =$
    c. $h\left(\frac{1}{2}\right) =$
    d. $g(-2) =$
    e. $f(x) + h(x) =$
    f. $g(x) - h(x) =$
    g. $f(h(1)) =$
    h. $h(g(-3)) =$
    i. $h(f(x)) =$
    j. $g(f(x)) =$

Algebra Review

Identify the following statements as true or false.

1. $\frac{2k}{2x+h} = \frac{k}{x+h}$ _____ 2. $\frac{1}{p+q} = \frac{1}{p} + \frac{1}{q}$ _____ 3. $\frac{x+y}{2} = \frac{x}{2} + \frac{y}{2}$ _____

4. $\frac{2k}{2x+h} = \frac{k}{x+h}$ _____ 5. $\frac{1}{p+q} = \frac{1}{p} + \frac{1}{q}$ _____ 6. $\frac{x+y}{2} = \frac{x}{2} + \frac{y}{2}$ _____

Identify the following statements as true or false over the set of real numbers. Give a counter example for any false statement.

7. $x^3 + 1 > x^3$ _____ 8. $x^3 + x > x^3$ _____ 9. $x^2 > 0$ _____

10. $x^2 \geq x$ _____ 11. $2x \geq x$ _____ 12. $\sqrt{x} \geq 0$ _____

11. $-x \leq 0$ _____ 14. $\frac{1}{x} \leq x$ _____ 15. $x \leq |x|$ _____

16. Solve $xy' + y + 1 = y' \; for \; y'$

17. Solve $\ln y = kt, \; for \; y.$

18. Factor: $y^3 + 27$

19. Factor: $x^2(x-1) - 4(x-1)$

Simplify each expression.

20. $\frac{(x^2)^3 x}{x}$ _____ 21. $\sqrt{x} \bullet \sqrt[3]{x} \bullet x^{\frac{1}{6}}$ _____

22. $\frac{5(x+h)^3 - 5x}{h}$ _____ 23. $\frac{3(x+h)^2 - 3x^2}{h}$ _____

24. $\frac{\frac{x^2-1}{x}}{\frac{x+1}{x}}$ _____ 25. $\frac{\frac{1}{x} + \frac{4}{x^2}}{3 - \frac{1}{x}}$ _____

26. $\frac{\frac{a}{2x+h} - \frac{a}{2x}}{h}$ _____ 27. $\frac{1}{1-2a} - \frac{2}{1+2a} + \frac{6a+2}{4a^2+1}$ _____

28. $\frac{x^2 - x + 7}{x}$ _____ 29. $\frac{8^{-2} \bullet 4^{-2}}{16^{-2}}$ _____

30. $\frac{3c^2 d^3}{(3cd^{-2})^2}$ _____ 31. $\sqrt{x} \bullet \sqrt[3]{x} \bullet \sqrt[4]{x}$ _____

Simplify, using factoring of binomial expressions. Leave answers in factored form.

Example:

$$\frac{(x+1)^3(4x-9)=(16x+9)(x+1)^2}{(x-6)(x+1)} = \frac{(x+1)^2[(x+1)(4x-9)-(16x+9)]}{(x-6)(x+1)}$$
$$= \frac{(x+1)^2(4x^2-5x-9-16x-9)}{(x-6)(x+1)}$$
$$= \frac{(x+1)^2(4x^2-21x-18)}{(x-6)(x+1)}$$
$$= \frac{(x+1)^2(4x+3)(x-6)}{(x-6)(x+1)}$$
$$= (x+1)(4x+3)$$

32. $\frac{(x-1)^2(3x-1)-2(x-1)\bullet 3}{(x-1)^4}$ _____

33. $(x-1)^3(2x-3)-(2x+12)(x-1)^2$ _____

34. $\frac{(x-1)^3(2x-3)-(4x-1)(x-1)^2}{(x-1)^2(2x-1)}$ _____

LOGARITHMS

Solve for x.

1. $\ln x = 3$
2. $log_{\frac{1}{2}} x = 4$
3. $log_3 81 = x$
4. $log_3 (-9) = x$
5. $log_x 16 = -4$
6. $log_x \frac{1}{25} = \frac{1}{2}$
7. $2{,}43e^x = 1.84$
8. $3e^{-5x} = 132$
9. $\ln(x+5) = -\ln(x-1) - \ln(x+1)$

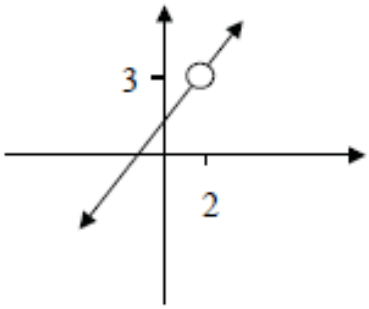
SOLVING EQUATIONS

SOLVE FOR x.

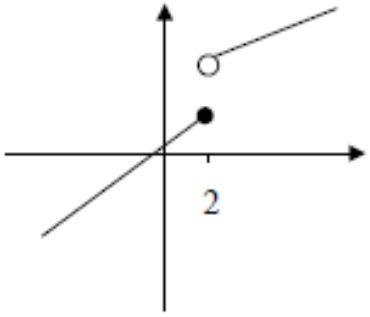
1. $4(x+3) - 3 = 2(4 - 3x) - 4$
2. $15 + x - 2x^2 = 0$
3. $5x^4 - 12x^3 = 0$
4. $\frac{1}{x-2} = 3$
5. $4t^3 - 12t^2 + 8t - 24 = 0$
6. $\frac{4}{x-3} - \frac{4}{x} = 1$
7. $\sqrt{x-2} - 8 = 0$
8. $(x+2)^{3/4} = 27$
9. $|x^2 - 6| = x$
10. $\left|x - \frac{3}{2}\right| \leq 1$
11. $x^2 - 2x - 8 < 0$
12. $e^{2x} - 7e^x + 10 = 0$
13. $ln\, x - ln\, 3 = 2$
14. $2ln\, 4x = 15$
15. $2cos\, x + 1 = 0$
16. $3sec^2 x - 4 = 0$
17. $sin^2 x = 3cos^2 x$
18. $\cos\frac{x}{2} = \frac{\sqrt{2}}{2}$
19. $log_2 x + log_2 5 = 6$
20. $3s^2 + 6s + 2 = 0$
21. $\frac{3x+5}{(x-1)(x^4+7)} = 0$

LIMITS

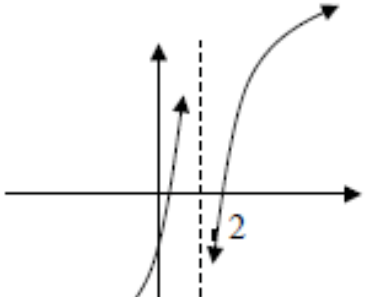
The limit of a function is the y-value that you are getting close to as *x* gets close to some number in the domain. In the "limit process" you never get to the limit, except for the limit of a constant function. We write $\lim_{x\to\infty} f(x)$, which is read "the limit of *f(x)* as *x* approaches a domain value of *a*." The limit must be the same as *x* approaches "*a*" from both the left and the right. To find the limit, substitute in values very close to "*a*" on both left and right and see if the y-value is approaching a single value. The limit does exist at a hole in a graph but does not exist at a vertical asymptote or a jump in the graph.



Limit exist at 2

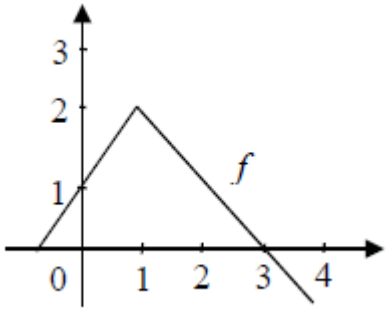


No limit at 2



The graphs of some functions are pictured below. Do you think that write $\lim_{x\to}$ t does exits, state its value. If not, then state why.

1.



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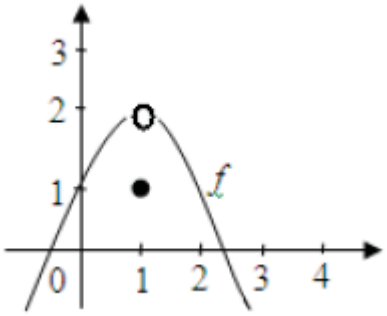

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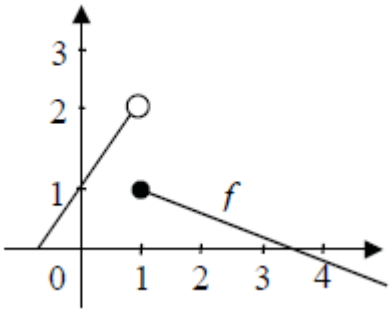

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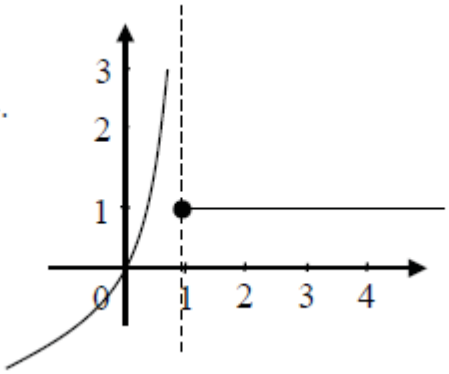

State the value of each of the following:

5. $\lim_{x\to 3} 5$

6. $\lim_{x\to -2} \frac{x^2-4}{x^3+8}$

7. $\lim_{x\to 1} \frac{x^2-1}{x-1}$

8. $\lim_{x\to 0} \frac{\sqrt{9+x}-3}{x}$

9. $\lim_{x\to 0} \frac{\sin x}{3x}$

10. $\lim_{x\to -2} |x|$

DERIVATIVES

Find the derivative using the definition of derivative. $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)=f(x)}{\Delta x}$

1. $f(x) = 2x$
2. $f(x) = x^2$
3. $f(x) = \frac{1}{x}$

Find the derivative using power rule.

4. $f(x) = x^3$
5. $f(x) = \sqrt{x}$
6. $f(x) = 6x^2 - 2x$
7. $f(x) = x^{\frac{4}{3}}$
8. $f(x) = 5x^{-4}$
9. $f(x) = \frac{1}{x^3}$

Find the derivative using Product Rule.

10. $f(x) = (2x^4 - 3)(x^2 + 1)$
11. $f(x) = (-3 + x^{-3})(-4x^3 + 3)$

Find the derivative using Quotient Rule.

12. $f(x) = \frac{5}{4x^3+4}$
13. $f(x) = \frac{4x^5+x^2+4}{5x^2-2}$

Find the derivative using Chain Rule.

14. $f(x) = (5x^2 + 3)^4$
15. $f(x) = \sqrt{-2x^2 + 1}$

COMPLEX NUMBERS

1.) *Draw the complex numbers in the coordinate system given by the modulus and argument.*

a) $|z|=1,\ \arg z=\pi$ b) $|z|=3,\ \arg z=\frac{5\pi}{6}$ c) $|z|=\frac{4}{5},\ \arg z=\frac{5\pi}{4}$

2.) *Find the following complex numbers in polar form.*

a) $z=2+2i$ b) $z=-4-3i$ c) $z=-1+i\sqrt{3}$

3.) *Write the following complex numbers in Cartesian form.*

a) z = 2 cis $\frac{\pi}{3}$ b) z =8 cis $\frac{11\pi}{6}$ c) z = $5e^{i\frac{3\pi}{4}}$ d) z = $e^{i\frac{13\pi}{4}}$

4.) *Multiply the complex numbers in Euler's or polar form. Give answer in the same form as the question.*
Arguments must be between 0 and 360^o when in degrees. Arguments must be between 0 and 2π when in radians.

a) $z_1=2e^{i\frac{\pi}{3}},\ z_2=4e^{i\frac{\pi}{4}}$ b) $z_1=5\,\text{cis}\ 90^o,\ z_2=6\,\text{cis}\ 45^o$

c) $z_1=\frac{2}{3}e^{i\frac{11\pi}{7}},\ z_2=\frac{5}{6}e^{i\frac{23\pi}{14}}$ d) $z_1=\frac{5}{6}\,\text{cis}\ 220^o,\ z_2=\frac{6}{5}\,\text{cis}\ 275^o$

5.) *Use De Moivre's theorem to calculate each of the following. Answers must be in simplified Cartesian form.*

a) $\left(2-2i\sqrt{3}\right)^5$ b) $(1+i)^{10}$ c) $\left(-3+i\sqrt{3}\right)^8$

6.) *Given the numbers* $z_1=e^{i\frac{\pi}{4}}$ *and* $z_2=\frac{\sqrt{3}}{2}+\frac{1}{2}i$ *find:*

a) z_1 in Cartesian form. b) z_2 in Euler's form c) $\frac{z_1}{z_2}$ in both forms. d) Hence, find the exact value of $\sin\left(\frac{\pi}{12}\right)$

e) Hence, find the exact value of $\cos\left(\frac{\pi}{12}\right)$ f) Hence find the exact value of $\tan\left(\frac{\pi}{12}\right)$

7.) *Given the numbers* $z_1=5\,cis\ 60^o$ *and* $z_2=3+3i$ *find:*

a) z_1 in Cartesian form. b) z_2 in polar form c) $\frac{z_1}{z_2}$ in both forms. d) Hence, find the exact value of $\sin\left(15^\circ\right)$

e) Hence, find the exact value of $\cos\left(15^\circ\right)$ f) Hence find the exact value of $\tan\left(15^\circ\right)$