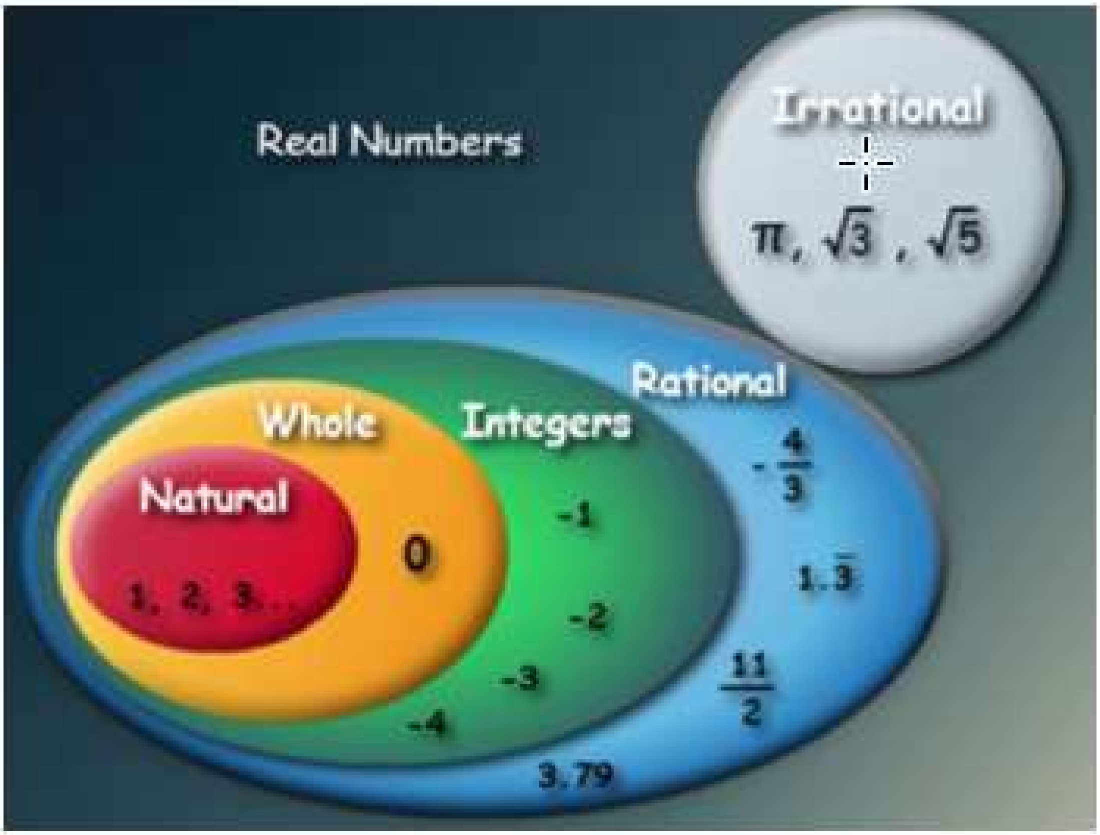
Topic 1: Basic Math Operations & Properties of Equality

The Real Number System

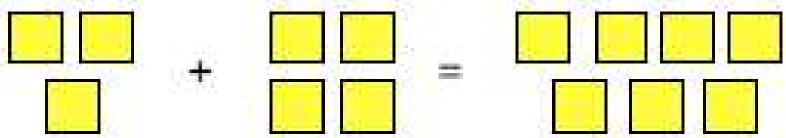


Adding and Subtracting Integers

The following squares will represent positive and negative units, respectively:

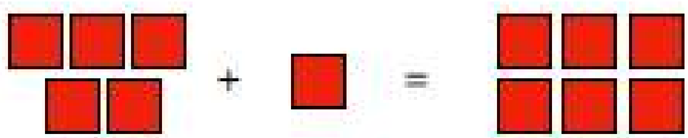
Use the squares to demonstrate.

Example 1: $3 + 4$



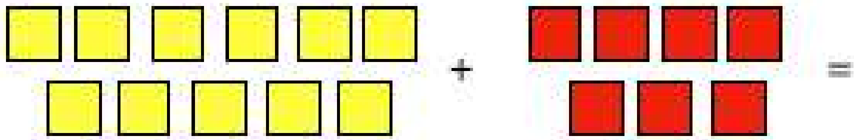
$3 + 4 = 7$

Example 2: $-5 + -1$

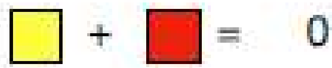


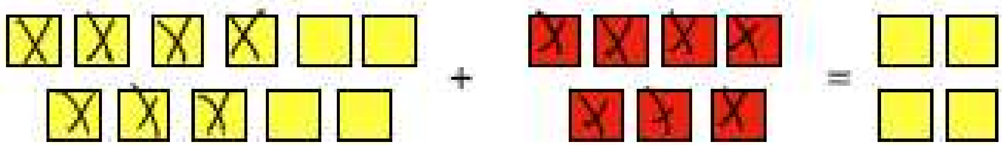
$-1 + -5 = -6$

Example 3: $11 + -7$



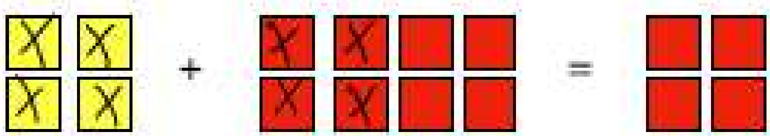
Special Note:





$11 + -7 = 4$

Example 4: $4 + (-8)$



$4 + (-8) = -4$

1) Provide a rule for addition both quantities are positive.

2) Provide a rule for addition both quantities are negative.

3) Provide a rule for addition when one quantity is negative and the other quantity is positive. What happens when there are more negative squares than positive squares? What happens when there are more positive squares than negative squares?

Practice Problems:

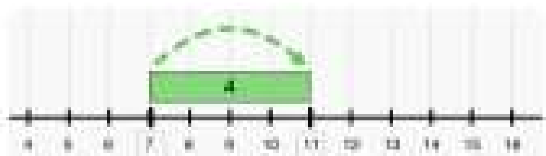
1) $4+9=$ 2) $-4+6=$ 3) $5+(-11)=$

4) $-7+7=$ 5) $-12+(-15)=$ 6) $32+(-13)=$

In previous classes, you learned to find the distance between two points a number line by using subtraction:

Example 5: $11 - 7 =$

The statement can be interpreted as the distance and direction of "7" to "11".

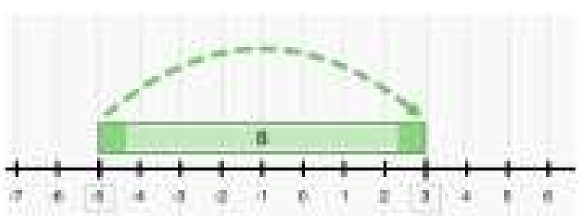


The distance of "7" to "11" is four units in a **positive** direction.

$11 - 7 = 4$

Example 6: $3 - (-5) =$

The statement can be interpreted as the distance and direction of "-5" to "3".

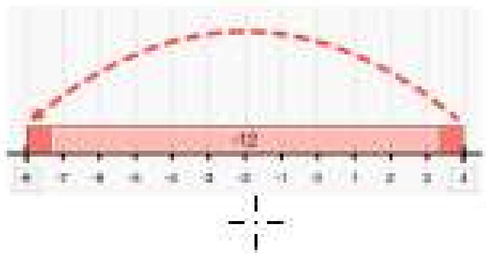


The distance of "-5" to "3" is eight units in a **positive** direction.

$5 - (-3) = 8$

Example 7: $-8-4=$

The statement can be interpreted as the distance and direction of "4" to "-8".

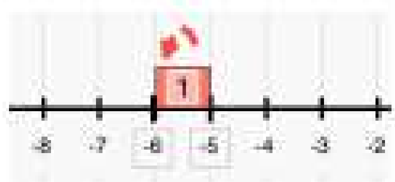


The distance of "4" to "-8" is 12 units in a **negative** direction.

$-8-4=-12$

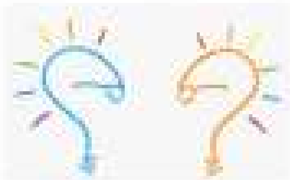
Example 7: $-6-(-5)=$

The statement can be interpreted as the distance and direction of "-5" to "-6".



The distance of "-5" to "-6" is one unit in a **negative** direction.

$-6-(-5)=-1$



1) Provide a general rule(s) when you have a "positive minus a negative".

2) Provide a general rule(s) when you have a "positive minus a positive".

3) Provide a general rule(s) when you have a "negative minus a positive".

4) Provide a general rule(s) when you have a "negative minus a negative".

Practice Set

1) $5-7=$ 2) $16-10=$ 3) $11-20=$

4) $-11-4=$ 5) $20-(-3)=$ 6) $-9-(-3)=$

7) $-4-(-4)=$ 8) $-9-2=$ 9) $-1-(-4)=$

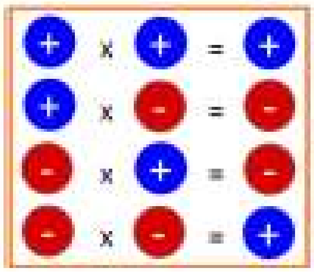
A negative sign , "-", changes a statement to its opposite meaning. Consider the following statement:

I do not want to not like you

The statement above means that I want to like you.

Whereas, the statement, "*I do not like you*" means that I have negative feelings towards you.

In mathematics, negative signs create opposite values and opposite statements. The following chart provides the rules for multiplying real numbers and dividing real numbers:



1) $2 \cdot 3 =$

2) $-8 \cdot -3 =$

3) $-11 \cdot 4 =$

4) $-10 \div 5 =$

5) $24 \div (-3)$

6) $44 \div 11 =$

7) $\dfrac{-60}{-5} =$

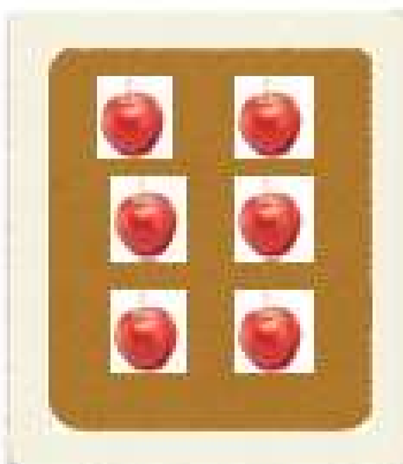
8) $-5 \cdot 3 \div (-5) =$

Algebraic Properties of Equality

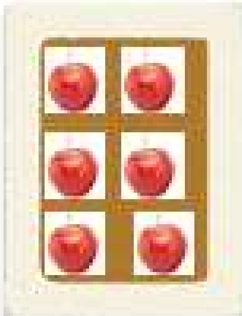
Properties of Equality	
Property	**Symbol**
Reflexive	$a = a$
Symmetric	If $a = b$ then $b = a$
Transitive	[illegible] and $b = c$ then $a = c$
Addition	If $a = b$ then $a + c = b + c$
Subtraction	If $a = b$ then $a - c = b - c$
Multiplication	If $a = b$ then $a \times c = b \times c$
Division	If $a = b$ then $a \div c = b \div c$
Substitution	If $a = b$ then b can replace a in any expression

The chart above provides different variables to represent any number in the real number system. Let's go use an apple festival to demonstrate each property:

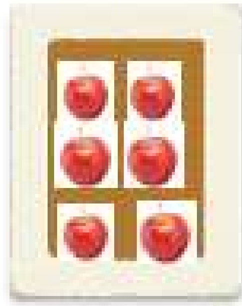
We will let the number of apples in the crate below represent "a" from the chart above.



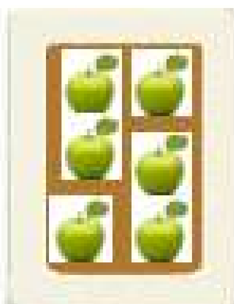
Reflexive Property of Equality $a = a$



=



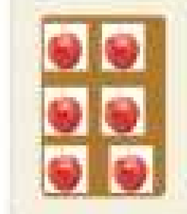
We will let the number of apples in the crate below represent "b" from the chart above.



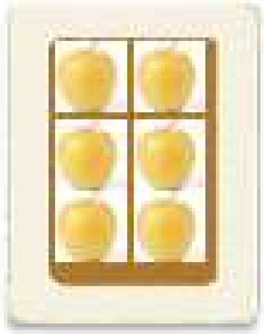
Symmetric Property of Equality $If\ a = b\ then\ b = a$.

=

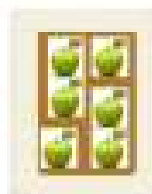
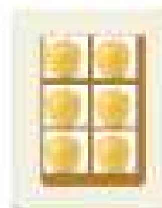
=



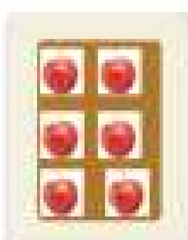
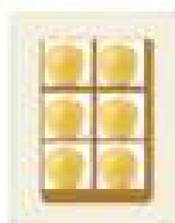
We will let the number of apples in the crate below represent “c” from the chart ab



<u>Transitive Property of Equality</u> If $a = b$ and $b = c$, then $a = c$.

 =  and  = 

then

 = 

Addition Property of Equality and Subtraction Property of Equality

We will change the definition of "c" to demonstrate the properties of equality for addition and subtraction. The variable "c" will represent a quantity of two apples of any type:

c = 2 apples of any type

<u>Addition Property of Equality</u>

If $a = b$ then $a + c = b + c$

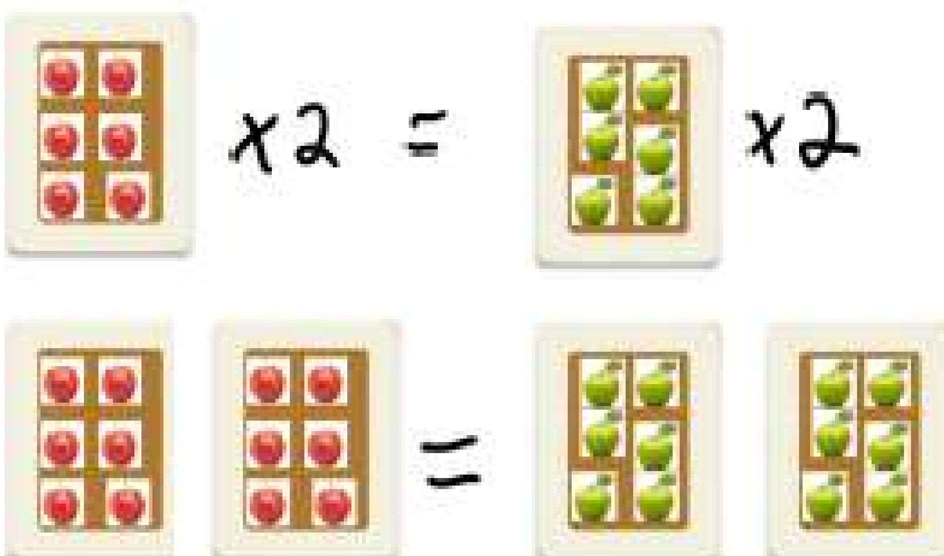
<u>Subtraction Property of Equality</u>

Multiplication Property of Equality

To demonstrate the multiplication property of equality, we will let “c” represent th
number of crates we want to purchase:

$$c = 2\ crates$$

If $a = b$ then $a \times c = b \times c$

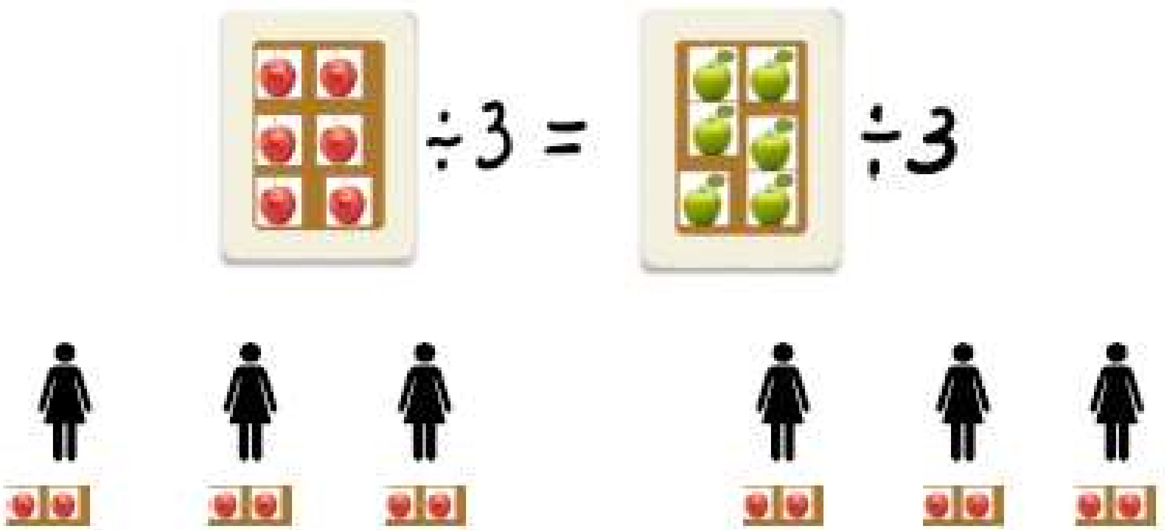


Division Property of Equality

If $a = b$ then $a \div c = b \div c$

To demonstrate the multiplication property of equality, we will let "c" represent the number of people who will receive an equal amount of apples from each crates **"a" and "b".**

$$c = 3\ people$$



Substitution Property of Equality

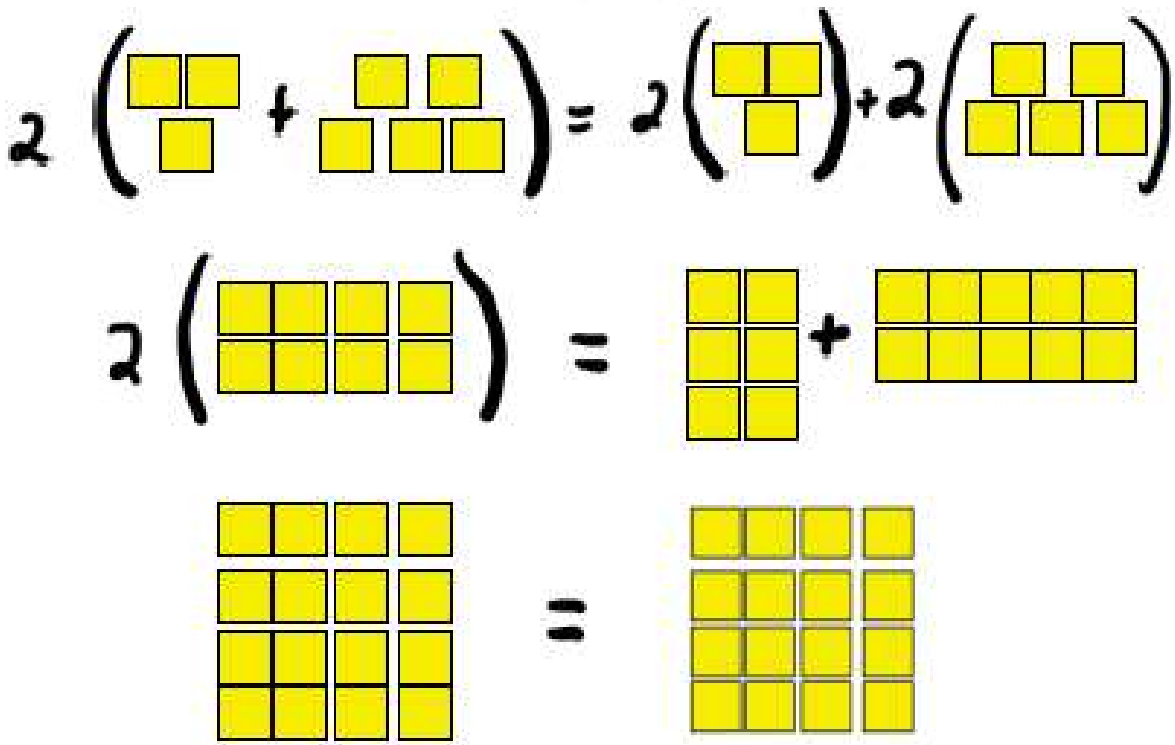
If $a = b$ then b can replace a in any expression

Distributive Property of Equality

$$a\,(b + c) = ab + ac$$

Let's use our squares to demonstrate the distributive property using the statement

$$2(3+5) = 2(3) + 2(5)$$



Properties of Real Numbers

Property	Addition	Multiplication
Commutative Property	$a+b=b+a$	$a \cdot b = b \cdot a$
Associative Property	$a+(b+c)=(a+b)+c$	$a \cdot (b \cdot c) = (a \cdot b) \cdot c$
Distributive Property	$a \cdot (b+c) = a \cdot b + a \cdot c$	
Identity Property	$a+0=a$	$a \cdot 1 = a$
Inverse Property	$a+(-a)=0$	$a \cdot \frac{1}{a} = 1$

Practice Set

Match the property with the correct description.

Place the letter besides the property that best fits the description.			
1)____	Additive Inverse Property	A	Changing the order in addition will not change the outcome as long as you move the sign in front with the number.
2)____	Multiplicative Inverse	B	I may reverse the sides of an equation as long as the signs stay with their respective numbers.
3)____	Distributive Property	C	Zero added to any number gives you that same number.
4)____	Commutative Property of Addition	D	Any number is equal to itself.
5)____	Commutative Property of Multiplication	E	I can replace any number with another number of equal value.
6)____	Properties of Equality	F	Any number multiplied by 1 gives you that same number.
7)____	Identity Property of Addition	G	Changing the order in multiplication does not change the outcome as long as you move the sign in front with the number.
8)____	Identity Property of Multiplication	H	Multiplying by the reciprocal equals one.
9)____	Reflexive Property of Equality	J	Opposites equal zero.
10)____	Symmetric Property of Equality	K	Involves both addition and subtraction. I can add first then multiply or multiply then add.
11)____	Substitution Property	L	I can add, subtract, multiply or divide both sides of an equation by the same number and still keep the equation balanced.

Simplifying Radicals

When we multiply a number times itself the product is a **perfect square:**

	Translation	
$1^2 = 1$	One "squared" equals one	1 x 1 = 1
$2^2 = 4$	Two "squared" equals four	2 x 2 = 4
$3^2 = 9$	Three "squared" equals nine.	3 x 3 = 9

The numbers **1, 4, and 9** are just three of an infinite set of perfect squares.

Practice: Provide three more perfect squares.

1)_________ **2)**_______ **3)**_______

When we multiply a number times itself **three times the** product is a **perfect cube:**

	Translation	
$1^3 = 1$	One "cubed" equals one	1 x 1 x 1 = 1
$2^3 = 8$	Two "cubed" equals four	2 x 2 x 2 = 8
$3^3 = 27$	Three "cubed" equals nine.	3 x 3 x 3 = 27

Practice: Provide three more perfect cubes.

1)_________ **2)**_______ **3)**_______

When we find the square root of a number, we are answering the question, "What number was squared to produce this number?"

Example 1: $\sqrt{36}$ What number was squared to produce "36" ?

$6^2 = 36$

Therefore, you will simplify by writing the solution:

$\sqrt{36} = 6$

Example 2: $\sqrt[3]{125}$ What number was cubed to produce "125"?

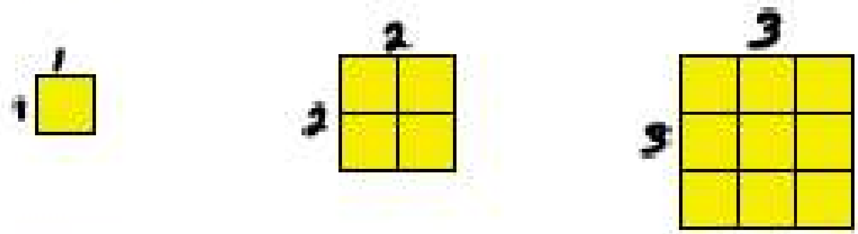
$5^3 = 125$

Therefore, you will simplify by writing the solution:

$\sqrt[3]{125} = 5$

You will be required to find the square root of a number that is not a perfect square. You will, also. Be required to find the cube root of a number that is not a perfect cube. In these cases, you must write the expression in **simplest radical form.**

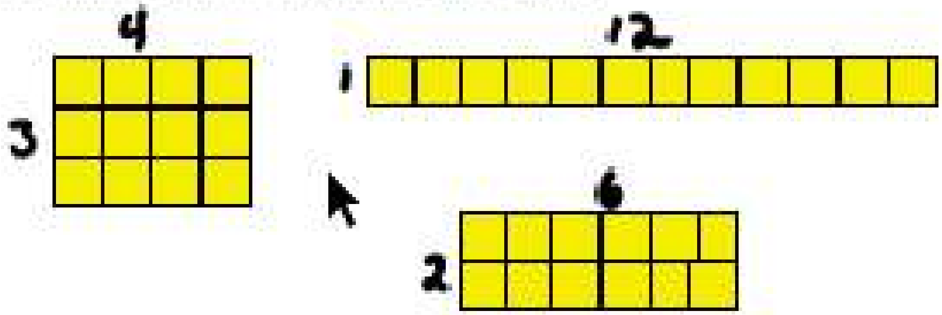
Recall that perfect squares are the product of a number being multiplied times itself. Geometrically, squares take this form:



1 square unit. 4 square unit 9 square unit

Example 3: $\sqrt{12}$

A square cannot be created with 12 square units:



Algebraically, we will find perfect squares in this figure and write $\sqrt{12}$ in simplest radical form. This will be accomplished by rewriting the expression under the radical as a product of prime factors:

$$\sqrt{12} = \sqrt{2 \cdot 2 \cdot 3}$$ → not a perfect square

perfect square
this product is "4"
$\sqrt{4} = 2$

$$\sqrt{12} = 2\sqrt{3}$$

Example 4: $\sqrt{40}$

$$\sqrt{40} = \sqrt{2 \cdot 2 \cdot 2 \cdot 5}$$ – not a perfect square

$$\sqrt{40} = 2\sqrt{10}$$

Example 4: $\sqrt[3]{40}$

$$\sqrt[3]{40} = \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 5}$$ → not a perfect cube

perfect cube $2^3 = 8$

$$\sqrt[3]{40} = 2\sqrt[3]{5}$$

Practice Set:

1) $\sqrt{0}$	2) $\sqrt{36}$	3) $\sqrt{72}$
4) $\sqrt{1}$	5) $\sqrt{40}$	6) $\sqrt{75}$
7) $\sqrt{4}$	8) $\sqrt{44}$	9) $\sqrt{76}$
10) $\sqrt{8}$	11) $\sqrt{45}$	12) $\sqrt{80}$
13) $\sqrt{9}$	14) $\sqrt{48}$	15) $\sqrt{81}$

16) $\sqrt[3]{0}$	17) $\sqrt[3]{243}$	18) $\sqrt[3]{72}$
19) $\sqrt[3]{1}$	20) $\sqrt[3]{432}$	21) $\sqrt[3]{75}$
22) $\sqrt[3]{4}$	23) $\sqrt[3]{250}$	24) $\sqrt[3]{54}$
25) $\sqrt[3]{8}$	26) $\sqrt[3]{45}$	27) $\sqrt[3]{80}$
28) $\sqrt[3]{9}$	29) $\sqrt[3]{1000}$	30) $\sqrt[3]{81}$

Topic 2: Translating Word Problems into Equations

Most of the time when someone says "word problems" there is automatic panic. But word problems do not have to be the worst part of a math class. By setting up a system and following it, you can be successful with word problems. What should you do? Here are some recommended steps:

1. Read the problem carefully and figure out what it is asking you to find.

Usually, but not always, you can find this information at the end of the problem.

2. Assign a variable to the quantity you are trying to find.

Most people choose to use x, but feel free to use any variable you like. For example, if you are being asked to find a number, some students like to use the variable n. It is your choice.

3. Write down what the variable represents.

At the time you decide what the variable will represent, you may think there is no need to write that down in words. However, by the time you read the problem several more times and solve the equation, it is easy to forget where you started.

4. Re-read the problem and write an equation for the quantities given in the problem.

This is where most students feel they have the most trouble. The only way to truly master this step is through lots of practice. Be prepared to do a lot of problems.

5. Solve the equation.

The examples done in this lesson will be linear equations. Solutions will be shown. However, they may not be as detailed as you would like.

Answer the question in the problem.

Just because you found an answer to your equation does not necessarily mean you are finished with the problem. Many times you will need to take the answer you get from the equation and use it in some other way to answer the question originally given in the problem.

7. Check your solution.

Your answer should not only make sense logically, but it should also make the equation true. If you are asked for a time value and end up with a negative number, this should indicate that you've made an error somewhere. If you are asked how fast a person is running and give an answer of 700 miles per hour, again you should be worried that there is an error. If you substitute these unreasonable answers into the equation you used in step 4 and it makes the equation true, then you should re-think the validity of your equation.

Let's Practice: Cover the answers and try to set up equation and then check yourself.

i. **When 6 is added to four times a number, the result is 50. Find the number.**

Step 1: What are we trying to find?
A number.

Step 2: Assign a variable for the number.
Let's call it *n*.

Step 3: Write down what the variable represents.
Let *n* = a number

Step 4: Write an equation.
We are told 6 is added to 4 times a number. Since n represents the number, four times the number would be 4*n*. If 6 is added to that, we get $6+4n$. We know that answer is 50, so now we have an equation $6+4n=50$

Step 5: Solve the equation.

$6+4n=50$

$4n=44$

$n=11$

Step 6: Answer the question in the problem
The problem asks us to find a number. We decided that *n* would be the number, so we have *n* = 11. The number we are looking for is 11.

Step 7: Check the answer.
The answer makes sense and checks in our equation from Step 4.

$$6+4(11)=6+44=50$$

ii. The sum of a number and 9 is multiplied by -2 and the answer is -8. Find the number.

Step 1: What are we trying to find?
A number.

Step 2: Assign a variable for the number.
Let's call it *n*.

Step 3: Write down what the variable represents.
Let *n* = a number

Step 4: Write an equation.
We know that we have the sum of a number and 9 which will give us *n* + 9. We are then told to multiply that by -2, so we have $-2(n+9)$. Be very careful with your parentheses here. The way this is worded indicates that we find the sum first and then multiply. We also know the answer is -8. So we will solve $-2(n+9)=-8$

Step 5: Solve the equation.

$$-2(n+9)=-8$$

$$-2n-18=-8$$

$$-2n=10$$

$$n=-5$$

Step 6: Answer the question in the problem
The problem asks us to find a number. We decided that *n* would be the number, so we have *n* = -5. The number we are looking for is -5.

Step 7: Check the answer.
The answer makes sense and checks in our equation from Step 4.

$$-2(n+9)=-2(-5+9)=-2(4)=-8$$

iii. On an algebra test, the highest grade was 42 points higher than the lowest grade. The sum of the two grades was 138. Find the lowest grade.

Step 1: What are we trying to find?
The lowest grade on an algebra test.

Step 2: Assign a variable for the lowest test grade.
Let's call it *l*.

Step 3: Write down what the variable represents.
Let *l* = the lowest grade

Step 4: Write an equation.
Whatever the lowest grade is, we are told that the highest grade is 42 points higher than that. That means we need to add 42 to the lowest grade. This tells us the highest grade is $l + 42$. We also know that the highest grade added to the lowest grade is 138. So, (highest grade) + (lowest grade) = 142. In terms of our variable, $(l+42)+(l)=138$

Step 5: Solve the equation.

$(l+42)+(l)=138$

$2l + 42 = 138$

$2l = 96$

$l = 48$

Step 6: Answer the question in the problem
The problem asks us to find the lowest grade. We decided that *l* would be the number, so we have *l* = 48. The lowest grade on the algebra test was 48.

Step 7: Check the answer.
The answer makes sense and checks in our equation from Step 4.

$(48+42)+(48)=90+48=138$

iv. At the end of the day, a pharmacist counted and found she has 4/3 as many prescriptions for antibiotics as she did for tranquilizers. She had 84 prescriptions for the two types of drugs. How many prescriptions did she have for tranquilizers?

Step 1: What are we trying to find?
The number of prescriptions for tranquilizers.

Step 2: Assign a variable for the number of tranquilizer prescriptions.
Let's call it *t*.

Step 3: Write down what the variable represents.
Let *t* = number of tranquilizer prescriptions

Step 4: Write an equation.
We have to be careful here. The pharmacist had 4/3 as many prescriptions for antibiotics as she did for tranquilizers. Let's think about this in terms of numbers first. Suppose there were 3 tranquilizer prescriptions, 4/3 as many would mean there were 4 prescriptions for antibiotics. Or if there were 30 tranquilizer prescriptions, then 4/3 as many for antibiotics, would mean there were 40 antibiotic prescriptions. In each case, we are taking the number of tranquilizers and multiplying by 4/3 to get the number of antibiotic prescriptions. So if *t* is the number of tranquilizer prescriptions, then $\frac{4}{3}t$ is the number of antibiotic prescriptions. We are told that together the two types of prescriptions add up to 84. So we end up with the equation $t + \frac{4}{3}t = 84$.

Step 5: Solve the equation.

$$t + \frac{4}{3}t = 84$$

$$\frac{7}{3}t = 84$$

$$t = 36$$

Step 6: Answer the question in the problem
The problem asks us to find the number of prescriptions for tranquilizers. We decided that *t* would be the number of prescriptions for tranquilizers, so we have *t* = 36. There were 36 prescriptions for tranquilizers.

Step 7: Check the answer.
The answer makes sense and checks in our equation from Step 4.

$$(36) + \frac{4}{3}(36) = 36 + 48 = 84$$

v. In a given amount of time, Jamie drove twice as far as Rhonda. Altogether they drove 90 miles. Find the number of miles driven by each.

Step 1: What are we trying to find?
The number of miles driven by Jamie and by Rhonda.

Step 2: Assign a variable.
Since we are looking for two numbers here, we need to choose which one we will assign a variable to. The number of miles driven by either Jamie or Rhonda will work. We need to just choose one and move to Step 3. Let's assign a variable to represent the number of miles driving by Rhonda

Let's call it *R*.

Step 3: Write down what the variable represents.
Let *R* = the number of miles driven by Rhonda

Step 4: Write an equation.
We know that Jamie drove twice as far a Rhonda. As with Example 4, let's think about this in terms of numbers before jumping into an equation. If Rhonda drives 10 miles, then Jamie will drive twice as far which would be 20. So whatever amount Rhonda drives, Jamie's amount will be two times that number. We have already decided that the number of miles driven by Rhonda is *R*, so the number of miles driven by Jamie is 2*R*. Together they drove a total of 90 miles. So we have (Rhonda) + (Jamie) = 90, or $R + 2R = 90$

Step 5: Solve the equation.

$$R + 2R = 90$$

$$3R = 90$$

$$R = 30$$

Step 6: Answer the question in the problem
The problem asks us to find out how far Rhonda and Jamie drove. The solution to the equation tells us *R* = 30, which means Rhonda drove 30 miles. Now we have to find out how far Jamie drove. She drove twice as far as Rhonda, so the distance would be 20 miles.

Step 7: Check the answer.
The answer makes sense and checks in our equation from Step 4.

$$(30)+2(30)=30+60=90$$

vi. **Karen works for $6 an hour. A total of 25% of her salary is deducted for taxes and insurance. She is trying to save $450 for a new car stereo and speakers. How many hours must she work to take home $450 if she saves all of her earnings?**

Step 1: What are we trying to find?
The number of hours Karen needs to work.

Step 2: Assign a variable for the number of hours.
Let's call it *h*.

Step 3: Write down what the variable represents.
Let *h* = the number of hours Karen needs to work

Step 4: Write an equation.
However many hours Karen works, we multiply that number by 6 to find out how much she earns. For example, if she worked, 10 hours, she would make $60 before taxes and insurance. So her salary before taxes and insurance will be 6*h*. From that amount, we have to subtract the amount taken out for taxes and insurance. 25% of her salary is taken away. We need to write 25% as a decimal which gives 0.25. But we have to take 25% OF her salary or 25% of 6*h*. Karen's goal is $450. We can now write an equation.

(Salary) - 25%(Salary) = 450

$$6h-.25(6h)=450$$

You may wonder why we did not use a dollar sign in the equation. Some students find the extra symbols distracting. It will be necessary to include dollars as part of any answer we may give involving money in this problem.

Step 5: Solve the equation.

$$6h-.25(6h)=450$$

$$6h-1.5h=450$$

$$4.5h=450$$

$$h=100$$

Step 6: Answer the question in the problem
The problem asks us to find how many hours Karen needs to work. We decided that h would be the number, so we have *h* = 100. Karen needs to work 100 hours to reach her goal of $450.

Step 7: Check the answer.

The answer makes sense and checks in our equation from Step 4.

$$6(100) - .25(6(100)) = 600 - 150 = 450$$

vii. The length of a rectangular map is 15 inches and the perimeter is 50 inches. Find the width.

Step 1: What are we trying to find?
The width of a rectangle.

Step 2: Assign a variable for the width.
Let's call it *w*.

Step 3: Write down what the variable represents.
Let *w* = the width of a rectangle

Step 4: Write an equation.
We know the length is 15 inches. We also know the perimeter is 50 inches. Perimeter is the distance all the way around a figure. So to go all the way around a rectangle, you have

Perimeter = width + length + width + length.

Since length is 15 inches, width is *w*, and perimeter is 50, we get

$$P = w + l + w + l \quad \text{or} \quad P = 2w + 2l$$

Step 5: Solve the equation.

$$P = w + l + w + l$$

$$50 = w + 15 + w + 15$$

$$50 = 2w + 30$$

$$20 = 2w$$

$$10 = w$$

Step 6: Answer the question in the problem.
The problem asks us to find the width of a rectangle. We decided that w would represent width, so we have *w* = 10. The width of the rectangle is 10 inches. Don't forget your units.

Step 7: Check the answer.
The answer makes sense and checks in our equation from Step 4.

$10 + 15 + 10 + 15 = 50$ inches

viii. The circumference of a circular clock face is 13.12 centimeters more than three times the radius. Find the radius of the face.

Step 1: What are we trying to find?
The radius of the face of a circular clock.

Step 2: Assign a variable for the radius.
Let's call it *r*.

Step 3: Write down what the variable represents.

Let *r* = the radius of the clock face

Step 4: Write an equation.
First we need to know a formula that will relate circumference and radius since those are two pieces of information in the problem. The formula for the circumference is $C = 2\pi r$. We are told that the circumference is 13.12 centimeters more than three times the radius. Three times the radius translates into 3*r*. Now we need to add 13.12 to that to get an expression for circumference.

$$C = 3r + 13.12$$

We now have two expressions for circumference. Since the circumference of a circle doesn't change, these two expressions must be equal. Now we can set up the equation

$$3r + 13.12 = 2\pi r$$

Step 5: Solve the equation.
In some classes your teacher may want you to leave π in its exact form rather than approximating the value as 3.14. We will use the approximation here. If your teacher wants you to leave π as part of your answer, you should ask how to do that.

$$3r + 13.12 = 2\pi r$$

$$3r + 13 = 2(3.14)r$$

$$3r + 13.12 = 6.28r$$

$$13.12 = 3.28r$$

$$4 = r$$

Step 6: Answer the question in the problem
The problem asks us to find the radius of the clock face. We decided that r would be the radius, so we have *r* = 4. The radius of the clock face is 4 centimeters. Don't forget your units.

Step 7: Check the answer.
The answer makes sense and checks in our equation from Step 4.

$$3(4) + 13.12 = 2(3.14)(4)$$

$$12 + 13.12 = (6.28)(4)$$

$$25.12 = 25.12$$

Practice Word Problems:

1. Twelve added to 3 times a number is 54. What is the number?

2. If 4 is subtracted from twice a number, the result is 10 less than the number. Find the number.

3. Twice a number is added to the number and the answer is 90. Find the number.

4. Jose has a board that is 44 inches long. He wishes to cut it into two pieces so that one piece will be 6 inches longer than the other. How long should the shorter piece be?

5. The sum of a number and 9 is multiplied by -2 and the answer is -8. Find the number.

6. On an algebra test, the highest grade was 36 points higher than the lowest grade. The sum of the two grades was 134. Find the lowest grade? Find the Highest Grade?

7. At the end of the day, a pharmacist counted and found she has 2/3 as many prescriptions for allergy medication as she did for asthma medication. She had 115 prescriptions for the two types of drugs. How many prescriptions did she have for asthma medication? How many for allergy medication?

8. In a given amount of time, James drove twice as far as Mac. Altogether they drove 351 miles. Find the number of miles driven by each.

9. Alaisha works for $9 an hour. A total of 23% of her salary is deducted for taxes and insurance. She is trying to save $885.60 for a new iPhone. How many hours must she work to earn $885.60 if she saves all her earnings?

10. The length of a rectangular map is 8 inches and the perimeter is 40 inches. Find the width.

11. The circumference of a circular clock face is 17.27 centimeters more than two times the radius. Find the radius of the face.

Answers:

1. n=14
2. n=-6
3. n=30
4. a=19
5. a=-5
6. lowest grade=49
   highest grade =85
7. 69 prescriptions for asthma drug
   46 prescriptions for allergy medicine
8. 117 miles for Mac and 234 miles for James
9. 80 hours
10. W=12
11. R=5.5

Topic 3:
Solving Equations: 1, 2 and Multi-step equations

Verifying Solutions to Equations

The solution of an equation is the replacement number for the variable that results in a true equation.

Example: $3x - 2 = 13$

In the replacement set {4,5,6}, which one is the solution?

3(**4**) – 2 = 13	3(**5**) – 2 = 13	3(**6**) – 2 = 13
12 – 2 = 13	15 – 2 = 13	18 – 2 = 13
10 = 13 False	13 = 13 ***True***	16 = 13 False

The solution is 5.

Practice:

Find the solution of each equation if the replacement **set is {-2, 0, 2}**

1) 5 + z = 3

2) 1 = 7z – (-1)

3) 8z + 5 = z – 9

Solving One-Step Equations (addition and subtraction)

You must get the variable (letter) by itself!!!!

Steps:

1) ***Isolate the variable by adding or subtracting whatever is with the variable to get rid of it!***
2) ***Check your solution***

Examples:

1) $m - 10 = 14$ (the opposite of subtraction is addition) **Check:**

$\underline{+10 \quad +10}$

$m = 24$ ← answer

2) $a + 9 = -3$ (the opposite of addition is subtraction) **Check:**

$\underline{-9 \quad -9}$

$a = -12$ ← answer

Practice:

1) $x + 15 = 18$

Check:

2) $n - 6 = -9$

Check:

3) $p - (-5) = 1$

Check:

4) -7 + w = -2

5) 38 = v + 11

6) y – (-6) = -7

Check :

Check :

Check :

<u>Solving One-Step Equations (multiplication and division)</u>

Steps:

1) ***Multiply or divide to get the variable by itself!! (Use the opposite operation)***
2) ***Check your solution***

Examples:

1. $\frac{2m}{2} = \frac{16}{2}$ *(the opposite of mult. is division)*

Check:

m = 8 ← answer

2. $\frac{a}{4}$ = -7 (the opposite of division is mult.)

Check:

$$\left(\frac{4}{1}\right)\frac{a}{4} = -7\left(\frac{4}{1}\right)$$

a = -28 ← answer

<u>Practice:</u> ***Don't forget to check your answers!!!***

1) 3x = 27

2) $\frac{n}{5} = -5$

3) $\frac{n}{-6} = -3$

4) -6w = -66

5) 6 = -10v

6) $-5 = \frac{x}{-6}$

To isolate a variable that is *multiplied by a fraction*, we move the fraction to the other side of the equation.

To move the fraction, we multiply by the **reciprocal (flip it !)** of the fraction.

Examples:

1). $\frac{2}{3}x$ = 9 (the opposite of multiplying by a fraction, is multiplying by the reciprocal).

$\left(\frac{3}{2}\right) \bullet \frac{2}{3}x = 9 \bullet \left(\frac{3}{2}\right)$ **Check:**

***x* = 13.5**

2). $-\frac{1}{4}m$ = 3 (the opposite of multiplying by a fraction, is multiplying by the reciprocal).

$(-4)\left(-\frac{1}{4}\right)m = 3(-4)$ **Check:**

m = -12

<u>Practice</u>: *Don't forget to check your answer!!!*

1). $\frac{1}{5}x = 6$

2). $-\frac{3}{4}x = 6$

3). $8 = \frac{2}{7}y$

4). $-24 = \frac{4}{7}y$

<u>Homework:</u>

Solve by isolating the variable. ***Check your solutions!!***

1. x + 8 = 12

2. x + 7 = 30

3. x – 13 = 44

4. x + 7 = -22

5. x – 15 = -34

6. x + 14 = 121

7. x – 63 = 17

8. x + 86 = 47

9. x + 9 = 25

10. x – 17 = 22

11. x + 11 = -23

12. x – 23 = -7

13. x – 9 = -27

14. x + 13 = -37

15. x + 22 = 11

16. $x + 29 = 4$

17. $x - 19 = -33$

18. $x - 9 = -27$

Multiplication and Division Equations:

1. $8y = 48$

2. $-6x = 42$

3. $3.6 = 12n$

4. $1.5 = -3w$

5. $0 = -0.6y$

6. $18 = 0.5x$

7. $-6 = -8y$

8. $\frac{7}{4}n = 1$

9. $\frac{1}{5}x = 10$

10. $\frac{3}{4}y = 12$

11. $-\frac{3}{5}y = -3$

12. $4 = -\frac{1}{2}x$

13. $8 = \frac{x}{-2}$

14. $\frac{4}{5}n = -\frac{1}{4}$

15. $-\frac{1}{2} = -\frac{5}{8}x$

16. $1.2y = 9$

17. $-4x = 12.8$

18. $-\frac{n}{3} = \frac{1}{3}$

Solving Two-Step Equations

Steps for isolating the variable:

1) ***Add or subtract first*** to remove anything that is with the variable
2) ***Multiply or divide second*** to solve for the variable

Example:

Problem: $3x - 8 = 7$ Check:

Step 1: $\underline{\quad + 8 \quad + 8}$ (Add 8)

Step 2: $\frac{3x}{3} = \frac{15}{3}$ Divide by 3

Solution: $x = 5$

Practice: Check your solutions!!!!

A) $12 = -3k + 3$

B) $5 - 2f = 19$

C) $-31 = -6w - 7$

D) $6 + 7r = 13$

E) $-8 = 8 - 2c$

F) $-4s + 1 = 9$

Solving Two-Step Equations with Like Terms

Steps for isolating the variable:

1. Simplify both sides of the equation by **Combining all Like Terms**
2. **Add or subtract first**, to move values to the opposite side of equation
3. **Multiply or divide second**, to finish moving values to opposite side

Example:

Problem: **2x – 17 + 5x = 4** **Check:**

Step 1: 7x – 17 = 4 Combine the 2x and 5x

Step 2: + 17 + 17 Add 17

Step 3: $\frac{7x}{7} = \frac{21}{7}$ Divide by 7

Solution: x = 3

Practice:

A) 19 = -3k + 3 – 5k

B) 5 – 2f + 4f = 23

C) -8= -6w +12 +2w

D) 3r -17 + 7r = 13

E) -8 = 8 – 2c + 7c

F) 9s - 4s + 1 = -9

<u>**Solving Two-Step Equations using the Distributive Property**</u>

Steps for isolating the variable to one side of the equation:

1) **Distribute:** Simplify both sides of the equation by distributing.
2) **Combine Like Terms**: Continue to simplify by combining any like terms.
3) **Add or subtract first**, to move values to the opposite side of equation.
4) **Multiply or divide second**, to finish moving values to opposite side.

Example:

Problem:	2(2x – 8) + 3x = 12		**Check:**
Step1:	4x – 16 + 3x = 12	Distribute the 2	
Step 2:	7x – 16 = 12	Combine the 4x and 3x	
Step 3:	+16 = + 16	Add 16	
Step 4:	$\frac{7x}{7} = \frac{28}{7}$	Divide by 7	
Solution:	x = 4		

Example:

Problem:	5(x + 3) – 2 = -7		**Check:**
Step 1:	5x + 15 – 2 = -7	Distribute the 5	
Step 2:	5x + 13 = - 7	Combine the 15 and (-2)	
Step 3:	- 13 = -13	Subtract 13	
Step 4:	$\frac{5x}{5} = \frac{-20}{5}$	Divide by 5	
Solution:	x = -4		

<u>Practice:</u>

1. 4(3x + 5) = 8	Check:	2. 3(x + 1) – 2 = 7	Check:
3. 3p – 8(1 + p) = 12	Check:	4. 6(2 + y) – 4 = -10	Check:

HOMEWORK

Solve and check your answers.

1. $2x - 5 = 11$
2. $5x - 7 = 13$
3. $4x + 1 = 9$
4. $8x + 3 = 19$
5. $-3x - 4 = 8$
6. $-6x + 3 = -9$
7. $2x + 25 = 13$
8. $14 - x = 22$
9. $2x + 7 = 13$
10. $-4x + 10 = 38$
11. $-12x - 17 = -89$
12. $6x + 14 = -64$
13. $13x - 29 = 153$
14. $7x + 15 = -20$
15. $16x - 55 = 41$
16. $45 - 13x = 58$

Solve and check your answers.

1. Combine Like Terms
2. Add or Subtract
3. Multiply or Divide

1. $16x + 42 - 13x = 24$

2. $23x - 14 - 7x = 82$

3. $14x + 43 - 25x = 219$

4. $-17x - 22 - 8x = 128$

5. $28x + 39 + 7x = 319$

6. $13x + 12 - 19x = -24$

7. $19x - 22 + 7x = 30$

8. $14x + 25 - 11x = 37$

9. $37x - 14 + 19x = 154$

10. $36x - 21 - 24x = -105$

11. $51x + 18 - 29x = -114$

12. $48x - 13 - 24x = 131$

13. $55x + 21 + 17x = 165$

14. $23x + 18 + 24x = -358$

Warm –up **Solving Equations using the Distributive Property**

1. $4(2x + 7) = 108$

2. $8(3x + 1) = 128$

3. $-6(5x + 2) = 198$

4. $8(-3x - 1) = -80$

5. $-7(4x - 7) = 105$

11. $3(4x - 7) = 135$

7. $10(5x - 3) = 20$

8. $7(2x - 4) = 28$

Solving Two-Step Equations with Variables on Both Sides

Steps for isolating the variable:

1). You must get the common variables to the same side of the equation by adding or subtracting the smaller of the two.

2). Add or subtract, to move values to the opposite side of equation.

3) Multiply or divide, to finish solving for the given variable.

Example: Problem: $3x - 20 = 4 - 5x$

$\underline{+5x \qquad\qquad +5x}$ Move the -5x over to the 3x and combine

Step 1: $8x - 20 = 4$

Step 2: $\underline{+20 \quad +20}$ Add 20

Step 3: $\frac{8x}{8} = \frac{24}{8}$ Divide by 8

Solution: $x = 3$

Check: $3x - 20 = 4 - 5x$

$3(3) - 20 = 4 - 5(3)$

$9 - 20 = 4 - 15$

$-11 = -11$ ✓

Example :

Problem: $8x + 25 = 3x$

$\underline{-3x \qquad -3x}$ Move the 3x over to the 8x and combine

Step 1: $5x + 25 = 0$

Step 2: $\underline{-25 \quad -25}$ Subtract 25

Step 3: $\frac{5x}{5} = \frac{-25}{5}$ Divide by 5

Solution: $x = -5$

Check: $8x + 25 = 3x$

$8(-5) + 25 = 3(-5)$

$-40 + 25 = -15$

$-15 = -15$ ✓

Practice:

1) $4x + 7 = 5x - 10$

2). $6y - 8 = -12 - 2y$

3). $3m + 4 = 8m + 19$

4). $5x + 12 = 9x$

5). $7n - 15 = 4n$

Classwork/Homework

1. $2x - 7 = 3x + 4$

2. $-7c + 9 = c + 1$

3. $4(2y - 4) = 5y + 2$

4. $-6 - (-2n) = 3n - (6 + 5)$

5. $4(t + 5) - 3 = 6t - 13$

6. $2(r - 4) = 5(r + - 7)$

11. $9a + 5 = 3a - 1$

12. $6(x - 9) = 4(x - 5)$

13. $2(x - 4) + 8 = 3x - 8$

14. $3x - 3 = -3x + -3$

15. $-10x + 6 = -7x + - 9$

16. $5 + 3x = 7(x + 3)$

7. 7 – 6a = 6 – 7a

17. $\frac{5}{2}$ x + 3 = $\frac{1}{2}$ x + 15

8. 12m – 9 = 4m + 15

18. 2x + 6 = 5x – 9

9. 8(x – 3) + 8 = 5x – 22

19. 4e – 19 = -(e + 4)

10. 3c – 12 = 14 + 5c

20. 5t + 7 = 4t – 9

<u>Rewriting Equations for a given Variable</u>

Rewrite in terms of the variable indicated:

1). D = rt, for t
formula for distance
solve for t, means get to t = ??????
since r is being multiplied by t,
divide r into both sides

$\frac{d}{r} = \frac{rt}{r}$ leaves

$\frac{d}{r} = t$

2. A = bh, for h
formula for area of a triangle
solve for h, means get to h = ???
since h is being multiplied by b,
divide both sides by b

$\frac{A}{b} = \frac{bh}{b}$ leaves

$\frac{A}{b} = h$

3. *i* = prt

Many times we solve for y:

formula for finding interest
solve for r
therefore, divide both sides by pt

$\frac{i}{pt} = \frac{prt}{pt}$ leaves

$\frac{i}{pt} = r$

4) 2m + y = r
solve for y: since adding 2m to y
subtract 2m from both sides

2m + y - 2m = r - 2m, leaves

y = r - 2m

Practice - Solve for y.

1) x - 2y = z

2) -2x - 3y = 10

3) $3x + \frac{2}{3}y = 15$

4) $x + \frac{y}{5} = 10$

<u>Homework</u>

1. a + b = c for b

2. x + y = z for y

3. 3t + u = v for u

4. x – y = z for x

5. x – y = z for y

6. 2x + y = z for x

7. 2m + y = r for y

8. a – b = c for a

9. a – b = c for b

10. st + u = v for t

Review for Test

Solve the equation, by getting the variable by itself.

1). x + 5 = 11

2). x – 8 = -14

3). $\frac{x}{5} = -2$

4). 6x = -24

5). $\frac{1}{3}x = 2$

6). $\frac{1}{3} = \frac{x}{18}$

7). 7x + 5 = 33

8). 45 = 20x - 35

9). 25 = $\frac{x}{9} + 13$

Solve the equations by combining like terms, then getting the variable by itself.

10). 4x + 2x = 12

11). –3x + x = -16

12). –7x +6 + 9x = 20

Solve the equations by putting the variables on the same side combining, then getting the variable by itself.

13). $3x = 2x + 2$

14). $27 - 4x = 5x$

15). $8x - 4 = 3(x - 2)$

<u>Classwork/Homework</u>

1) $x + 3 = 2$

2) $\frac{x}{2} = -6$

3) $3x = -12$

4) $2 + x = 7$

5) $-7x + 2 = 51$

6) $10 + x = 8$

7) $4x + 10 = 46$

8) $-3x + 7 = 7$

9) $\frac{24}{3} = 5x - x + 6$

10) $2x - 2 + 5x = 33$

11) $4 + 3x + 6x = 58$

12) $6x + 8 = 7x + 8$

13) $8 - 7x = x - 8$

14) $4 + 4x = x + 22$

15) $8 - 4x = -3x + \frac{56}{8}$

16) $-5(6x - 9) = \frac{76}{4}$

17) $-4(5 + 6x) = 244$

18) $2(4 - x) = -2$

19) $6(7 - 7x) = -378$

Solve for y:

20) $6x + y = 8$

Solve for y:

21) $5x - 10y = 15$

Solve for m:

22) $d = \frac{m}{V}$

Solve for n:

23) $PV = nRT$

Name KEY

Date ____________ Block ____________

Homework:

Solve by isolating the variable. _Check your solutions!!_

1. $x + 8 = 12$

$x = 4$

2. $x + 7 = 30$

$x = 23$

3. $x - 13 = 44$

$x = 57$

4. $x + 7 = -22$

$x = -29$

5. $x - 15 = -34$

$x = -19$

6. $x + 14 = 121$

$x = 107$

7. $x - 63 = 17$

$x = 80$

8. $x + 86 = 47$

$x = -39$

9. $x + 9 = 25$

$x = 16$

10. $x - 17 = 22$

$x = 39$

11. $x + 11 = -23$

$x = -34$

12. $x - 23 = -7$

$x = 16$

13. $x - 9 = -27$

$x = -18$

14. $x + 13 = -37$

$x = -50$

15. $x + 22 = 11$

$x = -11$

16. $x + 29 = 4$

$x = -25$

17. $x - 19 = -33$

$x = -14$

18. $x - 9 = -27$

$x = -18$

Multiplication and Division Equations:

1. $8y = 48$

$y = 6$

2. $-6x = 42$

$x = -7$

3. $3.6 = 12n$

$n = .3$

4. $1.5 = -3w$

$w = -.5$

5. $0 = -0.6y$

$\emptyset$

6. $18 = 0.5x$

$x = 36$

7. $-6 = -8y$

$y = \frac{3}{4}$

8. $\frac{7}{4}n = 1$

$n = \frac{4}{7}$

9. $\frac{1}{5}x = 10$

$x = 50$

10. $\frac{3}{4}y = 12$

$y = \frac{48}{3} = 16$

11. $-\frac{3}{5}y = -3$

$y = 5$

12. $4 = -\frac{1}{2}x$

$x = -8$

13. $8 = \frac{x}{-2}$

$x = -16$

14. $\frac{4}{5}n = -\frac{1}{4}$

$n = \frac{-5}{16}$

15. $-\frac{1}{2} = -\frac{5}{8}x$

$x = \frac{4}{5}$

16. $1.2y = 9$

$y = 7.5$

17. $-4x = 12.8$

$x = -3.2$

18. $-\frac{n}{3} = \frac{1}{3}$

$n = -1$

Date KEY Block ____________

Name ______________________________

HOMEWORK

Solve and check your answers.

1. $2x - 5 = 11$

   $x = 8$

2. $5x - 7 = 13$

   $x = 4$

3. $4x + 1 = 9$

   $x = 2$

4. $8x + 3 = 19$

   $x = 2$

5. $-3x - 4 = 8$

   $x = -4$

6. $-6x + 3 = -9$

   $x = 2$

7. $2x + 25 = 13$

   $x = -6$

8. $14 - x = 22$

   $x = -8$

9. $2x + 7 = 13$

   $x = 3$

10. $-4x + 10 = 38$

    $x = -7$

11. $-12x - 17 = -89$

    $x = 6$

12. $6x + 14 = -64$

    $x = -13$

13. $13x - 29 = 153$

    $x = 14$

14. $7x + 15 = -20$

    $x = -5$

15. $16x - 55 = 41$

    $x = 6$

16. $4(2x + 7) = 108$

    $8x + 28 = 108$

    $-28 \quad -28$

    $8x = 80$

    $x = 10$

17. $8(3x + 1) = 128$

    $x = 5$

18. $-6(5x + 2) = 198$

    $x = -7$

19. $8(-3x - 1) = -80$

    $x = 3$

20. $-7(4x - 7) = 105$

    $x = -2$

21. $3(4x - 7) = 135$

    $x = 13$

Homework

Name KEY

Date ______________ Block ______

Solve and check your answers.

1. Combine Like Terms
2. Add or Subtract
3. Multiply or Divide

1. $16x + 42 - 13x = 24$

$x = -6$

2. $23x - 14 - 7x = 82$

$x = 6$

3. $14x + 43 - 25x = 219$

$x = -16$

4. $-17x - 22 - 8x = 128$

$x = -6$

5. $28x + 39 + 7x = 319$

$x = 8$

6. $13x + 12 - 19x = -24$

$x = 6$

7. $19x - 22 + 7x = 30$

$x = 2$

8. $14x + 25 - 11x = 37$

$x = 4$

9. $37x - 14 + 19x = 154$

$x = 3$

10. $36x - 21 - 24x = -105$

$x = -7$

11. $51x + 18 - 29x = -114$

$x = -6$

12. $48x - 13 - 24x = 131$

$x = 6$

13. $55x + 21 + 17x = 165$

$x = 2$

14. $23x + 18 + 24x = -358$

$x = -8$

Solving Equations using the Distributive Property

1). $-5 = 3(2x + 1) + 7x$

$-5 = 6x + 3 + 7x$

$-5 = 13x + 3$

$-3 \quad -3$

$\frac{-8}{13} = \frac{13x}{13}$

$x = -\frac{8}{13}$

2). $-40 = 2(2y - 6) + 4y$

$-40 = 4y - 12 + 4y$

$-40 = 8y - 12$

$+12 \quad +12$

$\frac{-28}{8} = \frac{8y}{8}$

$-\frac{7}{2} = y$

3). $11m - (6m - 5) = 25$

$11m - 6m + 5 = 25$

$5m + 5 = 25$

$-5 \quad -5$

$5m = 20$

$m = 4$

4). $2(4x - 5) - 4 = 20$

$8x - 10 - 4 = 20$

$8x - 14 = 20$

$+14 \quad +14$

$\frac{8x}{8} = \frac{34}{8} \div \frac{2}{2}$

$x = \frac{17}{4}$

5). $-15 = 5(3q - 10) - 5q$

$-15 = 15q - 50 - 5q$

$-15 = 10q - 50$

$+50 \quad +50$

$35 = 10q$

$\frac{5 \div 35}{5 \div 10} = q$

$q = \frac{7}{2}$

6). $x - 2(x + 10) = 12$

$x - 2x - 10 = 12$

$x - 10 = 12$

$+10 \quad +10$

$x = 22$

7). $-7m + 2(5m + 4) = 17$

$-7m + 10m + 8 = 17$

$3m + 8 = 17$

$-8 \quad -8$

$\frac{3m}{3} = \frac{9}{3}$

$m = 3$

8). $4(h + 2) - 3h = -6$

$4h + 8 - 3h = -6$

$h + 8 = -6$

$-8 \quad -8$

$h = -14$

Homework - 2-Step Equations

Variables on Both Sides

Name: KEY

Date: *Block:*

---

1. $2x - 7 = 3x + 4$

$x = -11$

2. $-7c + 9 = c + 1$

$c = 1$

3. $4(2y - 4) = 5y + 2$

$y = 6$

4. $-6 - (-2n) = 3n - (6 + 5)$

$n = 5$

5. $4(t + 5) - 3 = 6t - 13$

$t = 15$

11. $9a + 5 = 3a - 1$

$a = -1$

12. $6(x - 9) = 4(x - 5)$

$x = 17$

13. $2(x - 4) + 8 = 3x - 8$

$x = 8$

14. $3x - 3 = -3x + -3$

No solution

15. $-10x + 6 = -7x + -9$

$x = 5$

6. $2(r-4)=5(r+-7)$

$r=9$

7. $7-6a=6-7a$

$a=-1$

8. $12m-9=4m+15$

$m=3$

9. $8(x-3)+8=5x-22$

$x=-2$

10. $3c-12=14+5c$

$c=-13$

16. $5+3x=7(x+3)$

$x=-4$

17. $\frac{5}{2}x+3=\frac{1}{2}x+15$

$x=6$

18. $2x+6=5x-9$

$x=5$

19. $4e-19=-(e+4)$

$e=3$

20. $5t+7=4t-9$

$t=-16$

Name KEY

Date ___________ Block _____

Homework

1. $a + b = c$ for b

$b = -a + c$

2. $x + y = z$ for y

$y = -x + z$

3. $3t + u = v$ for u

$u = -3t + v$

4. $x - y = z$ for x

$x = y + z$

5. $x - y = z$ for y

$y = x - z$

6. $2x + y = z$ for x

$x = -\frac{y + z}{2}$

7. $2m + y = r$ for y

$y = -2m + r$

8. $a - b = c$ for a

$a = b + c$

9. $a - b = c$ for b

$b = a - c$

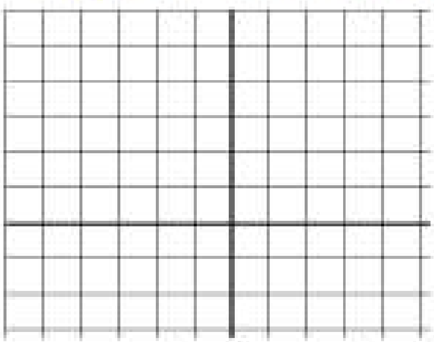
10. $st + u = v$ for t

$t = \frac{v - u}{s}$

Finding Slope from Two Points

$$\text{slope} = \frac{\textit{change in } y}{\textit{change in } x} \quad \text{or} \quad \frac{\textit{rise}}{\textit{run}}$$

Given two points (-2, 3) and (4, 6), think of some ways to find the change in y values from point to point, and the change in x values from point to point.



slope Formula: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

ex.1) Find the slope between each set of points.

a. (3, -4), (-2, 5)

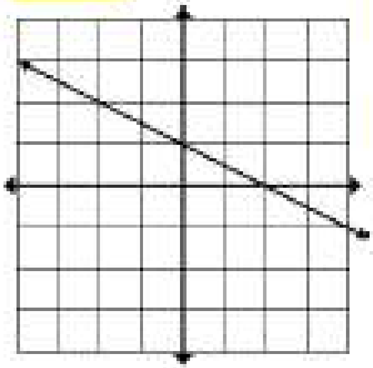
b. (2, 5), (2, -1)

c. (4, -3), (3, -3)

Write the Equation of a Graph Steps:

1. Find the y-intercept.
2. Calculate the slope.
3. Use y=mx+b: replace m with slope, and b with y-intercept.

ex.3)

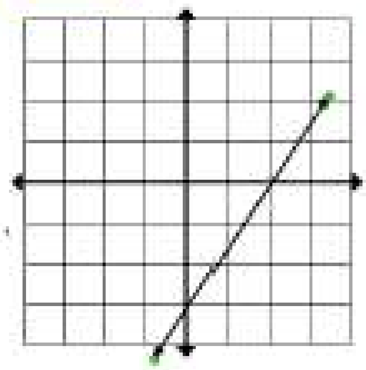


m = ______

b = ______

y = ____________________

ex.4)



m = ______

b = ______

y = ____________________

Point-Slope Form &
Finding Equation of a Line Given Two Points

Point-Slope Form

slope

$$y - y_1 = m(x - x_1)$$

coordinates of a point on the line

Example 3: Writing Linear Equations in Slope-Intercept Form

Write an equation in slope-intercept form for the line with slope 3 that contains (–1, 4).

Step 1 Write the equation in point-slope form:

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 3[x - (-1)]$$

Step 2 Write the equation in slope-intercept form by solving for y.

$$y - 4 = 3(x + 1)$$ *Rewrite subtraction of negative numbers as addition.*

$$y - 4 = 3x + 3$$ *Distribute 3 on the right side.*

$$+4 \qquad +4$$ *Add 4 to both sides.*

$$\boxed{y = 3x + 7}$$

Finding Equation of Line Given Two Points(Method 1)

Write an equation in slope-intercept form for the line through the two points.

A $(1, -4)$ **and** $(3, 2)$

Step 1 Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-4)}{3 - 1} = \frac{6}{2} = 3$$

Step 2 Substitute the slope and one of the points into the point-slope form.

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 3(x - 3)$$ *Choose (3, 2).*

Step 3 Write the equation in slope-intercept form.

$$y - 2 = 3(x - 3)$$

$$y - 2 = 3x - 9$$

$$\underline{+2} \quad \underline{+2}$$

$$\boxed{y = 3x - 7}$$

B $(4, -7)$ **and** $(0, 5)$

Step 1 Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} =$$

Step 2 Substitute the slope and one of the points into the point-slope form.

$$y - y_1 = m(x - x_1)$$

Step 3 Write the equation in slope-intercept form.

Finding Equation of Line Given Two Points (Method 2)

Write the equation of the line that goes through the points (1, 6) and (3, -4)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{-4 - 6}{3 - 1} \quad \frac{-10}{2} \quad -5$$

$Y = mX + b$

$6 = (-5)(1) + b$

$6 = -5 + b$

$+5 \quad +5$

$11 = b$

$Y = -5X + 11$

Try Using Both Methods:

Find the equation of the line in Slope-Intercept Form that passes through (2, 5) and (3, 9).

Solve Using Method 1	**Solve Using Method 2**

Find the slope of the line through each pair of points.

1. $(-16, 7), (-15, 17)$

2. $(16, 1), (17, 7)$

Find the slope and y-intercept of each equation.

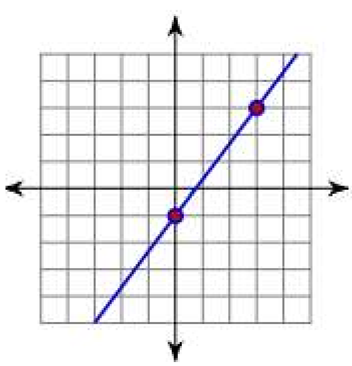
3. $2y - 10 = -4x$

Write an equation in the slope-intercept form

4.

through: $(-2, -1)$ and $(-4, -3)$

5.

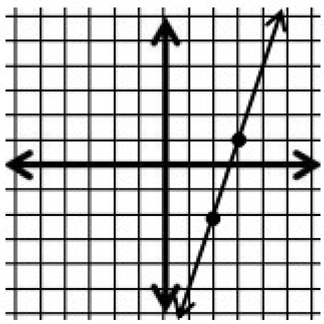


Write an equation in point-slope form of the line that passes through the given point and has the given slope

6.

(-6, -2); m = 3

7.



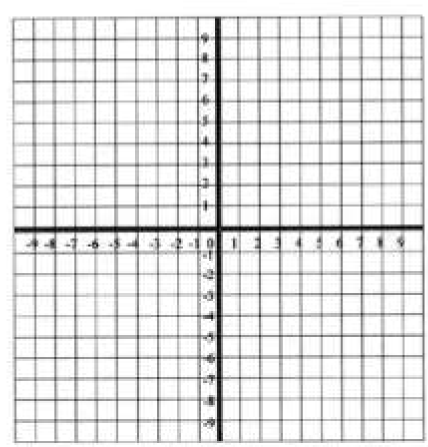
8. through

(4,7) and (5, 1)

Find the x- and y-intercepts of each equation and then graph the line.

9.

$3x - y = 9$



x-int = ______ y-int = ________

Write each equation in standard form using integers

10.

$y = -3x$

11.

Write an equation of a line (in standard form) that has the same slope as the line $3x - 5y = 7$ and the same y-intercept as the line $2y - 9x = 8$.

Write an equation of a line that is parallel and an equation of a line that perpendicular to the given information

12.

Slope $= -\frac{4}{3}$, y-intercept = 1

Parallel:____________________________

Perpendicular: ___________________________

Topic 5: Systems of Equations

NOTES – SYSTEMS OF LINEAR EQUATIONS

System of Equations – a set of equations with the same variables
(two or more equations graphed in the same coordinate plane)

Solution of the system – an ordered pair that is a solution to all equations
is a solution to the equation.

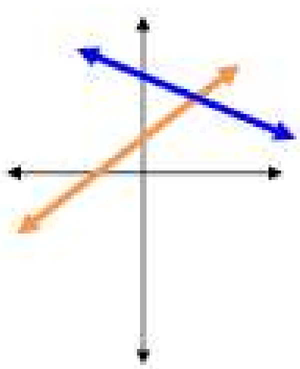
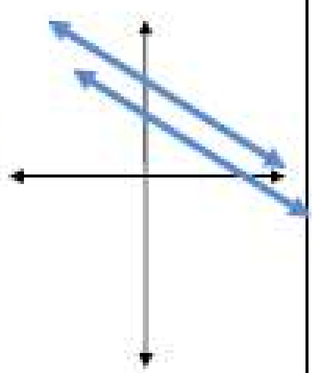
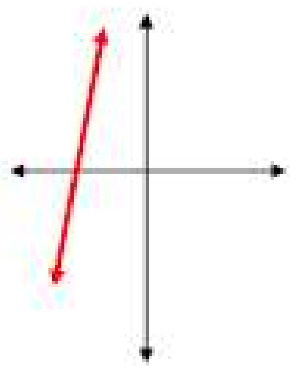
a. one solution
b. no solution
c. an infinite number of solutions

Other terminology

consistent – a system that has at least one solution

a. **independent** – has exactly one solution
b. **dependent** – an infinite number of solutions

inconsistent – a system that has no solution

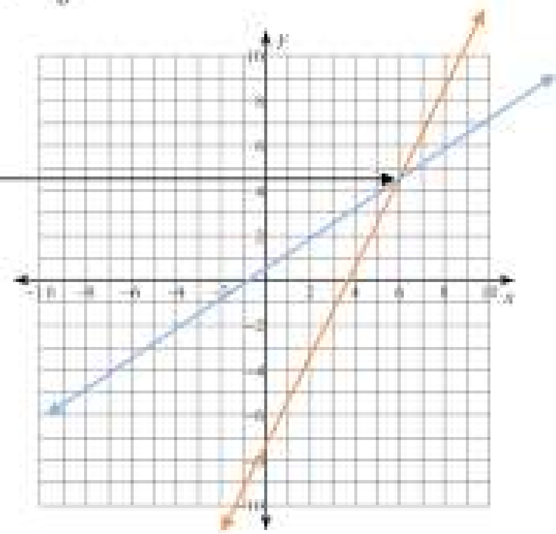
Number of Solutions (solutions are where they intersect)	exactly one solution	no solution	Infinitely many solutions
Definitions	consistent and independent	inconsistent	consistent and dependent
Graph			

****To solve a system of equations by graphing simply graph both equations on the same coordinate plane and find where they intersect.*

THREE METHODS FOR SOLVING SYSTEMS OF EQUATIONS

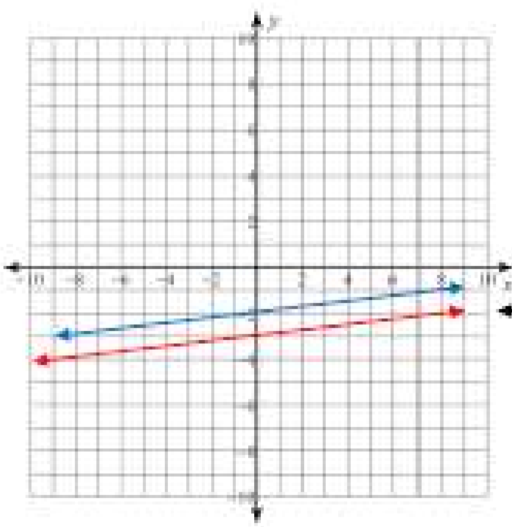
1. **Graphing**
   a. Graph one equation $y = 2x - 7$
   b. Graph the other equation on the same plane. $y = \frac{2}{3}x + 1$
   c. Find the point, or points, or intersection.

Ex 1:



(6, 5) is the solution to the system.
It is consistent and independent.

Ex 2: $x - 27 = 9y$
$18 = x - 9y$



$x - 27 = 9y$

$\frac{x}{9} - \frac{27}{9} = \frac{9y}{9}$

$\frac{1}{9}x - 3 = y$

$18 = x - 9y$
$\underline{-x} \quad \underline{-x}$
$-x + 18 = -9y$

$\frac{-x}{-9} + \frac{18}{-9} = \frac{-9y}{-9}$

$\frac{1}{9}x - 2 = y$

The lines are parallel.
There is no solution to this system.
It is inconsistent.

Ex 3: $2x + 3y = 15$ and $y = -\frac{2}{3}x + 5$

$\underline{-2x} \qquad \underline{-2x}$
$3y = -2x + 15$

$\frac{3y}{3} = \frac{-2x}{3} + \frac{15}{3}$

$y = \frac{-2}{3}x + 5$

They are the same equation so they would graph into the same line.

There are infinitely many solutions.

This system is consistent and dependent.

2. Substitution

a. If possible, solve at least one equation for one variable.
b. Substitute the result into the other equation to replace one of the variables.
c. Solve the equation.
d. Substitute the value you just found into the first equation.
e. Solve for the other variable.
f. Write the solution as an ordered pair.

I'm choosing to solve for this y.

{Quick steps: Solve, Substitute, Solve, Substitute, Solve, Write the solution}

Ex 1: $8x + 5y = 2$ *and* $-2x + y = 4$

$$-2x + y = 4$$
$$\underline{+2x \qquad +2x}$$
$$y = 2x + 4$$

$$8x + 5(2x + 4) = 2$$
$$8x + 10x + 20 = 2$$
$$18x + 20 = 2$$
$$\underline{-20 \quad -20}$$
$$18x = -18$$
$$\frac{18x}{18} = \frac{-18}{18}$$
$$x = -1$$

$$y = 2(-1) + 4$$
$$y = -2 + 4$$
$$y = 2$$

The solution is $(-1,\ 2)$.
It is consistent and independent.

Ex 2: $-2x + 2y = -4$ *and* $-x = -y - 4$ *(let's solve this one for x first)*

$$\frac{-x}{-1} = \frac{-y}{-1} - \frac{4}{-1}$$
$$x = y + 4$$

$$-2(y + 4) + 2y = -4$$
$$-2y - 8 + 2y = -4$$
$$-2y - 8 + 2y = -4$$
$$-8 = -4$$

This is a false statement, therefore this system has no solution.
The lines are parallel and are inconsistent.

Ex 3: $2x + y = 5$ *and* $-6x - 3y = -15$

$$2x + y = 5$$
$$\underline{-2x \qquad -2x}$$
$$y = -2x + 5$$

$$-6x - 3(-2x + 5) = -15$$
$$-6x + 6x - 15 = -15$$
$$-6x + 6x - 15 = -15$$
$$-15 = -15$$

This is a true statement, therefore this system has infinitely many solutions. It is consistent and dependent.

3. **Elimination**

a. Write both equations so that like terms are aligned vertically.
b. Multiply one or both equations by a constant to get two equations that contain at least one set of exactly opposite terms.
c. Add the equations, eliminating one variable.
d. Solve for the remaining variable.
e. Substitute the value from (d) into one of the equations and solve for the other variable.
f. Write the solution as an ordered pair.

Ex 1: $6x + 14y = 6 \rightarrow 6x + 14y = 6 \rightarrow 6x + 14y = 6$

$-4x - 7y = -11 \rightarrow 2[-4x - 7y = -11] \rightarrow \underline{-8x - 14y = -22}$

$-2x = -16$

$\frac{-2x}{-2} = \frac{-16}{-2}$

$x = 8$

I choose the first equation to substitute x = 8 into.

$6(8) + 14y = 6$

$48 + 14y = 6$

$\underline{-48} \quad \underline{-48}$

$14y = -42$

$\frac{14y}{14} = \frac{-42}{14}$

$y = -3$

The solution is (8, −3). It is consistent and independent.

Ex 2: $-8x + 2y = -10 \rightarrow -8x + 2y = -10 \rightarrow -8x + 2y = -10$

$-4x + y = -2 \rightarrow -2[-4x + y = -2] \rightarrow \underline{8x - 2y = 4}$

$0 = -6$

This is a false statement and has no solution. The lines are parallel. It is inconsistent.

Ex 3: $6x + 8y = -28 \rightarrow 6x + 8y = -28 \rightarrow 6x + 8y = -28$

$-3x - 4y = 14 \rightarrow 2[-3x - 4y = 14] \rightarrow \underline{-6x - 8y = 28}$

$0 = 0$

This is a true statement and has infinitely many solutions. The equations are the exact same line. It is consistent and dependent.

Topic 5: Systems of Equations

Solving Systems of Equations by Substitution

Date________________ Period____

Solve each system by substitution.

1) $y = 6x - 11$
$-2x - 3y = -7$

2) $2x - 3y = -1$
$y = x - 1$

3) $y = -3x + 5$
$5x - 4y = -3$

4) $-3x - 3y = 3$
$y = -5x - 17$

5) $y = -2$
$4x - 3y = 18$

6) $y = 5x - 7$
$-3x - 2y = -12$

7) $-4x + y = 6$
$-5x - y = 21$

8) $-7x - 2y = -13$
$x - 2y = 11$

9) $-5x + y = -2$
$-3x + 6y = -12$

10) $-5x + y = -3$
$3x - 8y = 24$

Solutions:

Solve each system by substitution.

1) $y = 6x - 11$
$-2x - 3y = -7$
$(2, 1)$

2) $2x - 3y = -1$
$y = x - 1$
$(4, 3)$

3) $y = -3x + 5$
$5x - 4y = -3$
$(1, 2)$

4) $-3x - 3y = 3$
$y = -5x - 17$
$(-4, 3)$

5) $y = -2$
$4x - 3y = 18$
$(3, -2)$

6) $y = 5x - 7$
$-3x - 2y = -12$
$(2, 3)$

7) $-4x + y = 6$
$-5x - y = 21$
$(-3, -6)$

8) $-7x - 2y = -13$
$x - 2y = 11$
$(3, -4)$

9) $-5x + y = -2$
$-3x + 6y = -12$
$(0, -2)$

10) $-5x + y = -3$
$3x - 8y = 24$
$(0, -3)$

Solving Systems of Equations by Elimination

Date________________ Period____

Solve each system by elimination.

1) $-4x - 2y = -12$
   $4x + 8y = -24$

2) $4x + 8y = 20$
   $-4x + 2y = -30$

3) $x - y = 11$
   $2x + y = 19$

4) $-6x + 5y = 1$
   $6x + 4y = -10$

5) $-2x - 9y = -25$
   $-4x - 9y = -23$

6) $8x + y = -16$
   $-3x + y = -5$

7) $-6x + 6y = 6$
   $-6x + 3y = -12$

8) $7x + 2y = 24$
   $8x + 2y = 30$

9) $5x + y = 9$
   $10x - 7y = -18$

10) $-4x + 9y = 9$
    $x - 3y = -6$

11) $-3x + 7y = -16$
    $-9x + 5y = 16$

12) $-7x + y = -19$
    $-2x + 3y = -19$

Solutions:

Solve each system by elimination.

1) $-4x - 2y = -12$
$4x + 8y = -24$

(6, −6)

2) $4x + 8y = 20$
$-4x + 2y = -30$

(7, −1)

3) $x - y = 11$
$2x + y = 19$

(10, −1)

4) $-6x + 5y = 1$
$6x + 4y = -10$

(−1, −1)

5) $-2x - 9y = -25$
$-4x - 9y = -23$

(−1, 3)

6) $8x + y = -16$
$-3x + y = -5$

(−1, −8)

7) $-6x + 6y = 6$
$-6x + 3y = -12$

(5, 6)

8) $7x + 2y = 24$
$8x + 2y = 30$

(6, −9)

9) $5x + y = 9$
$10x - 7y = -18$

(1, 4)

10) $-4x + 9y = 9$
$x - 3y = -6$

(9, 5)

11) $-3x + 7y = -16$
$-9x + 5y = 16$

(−4, −4)

12) $-7x + y = -19$
$-2x + 3y = -19$

(2, −5)

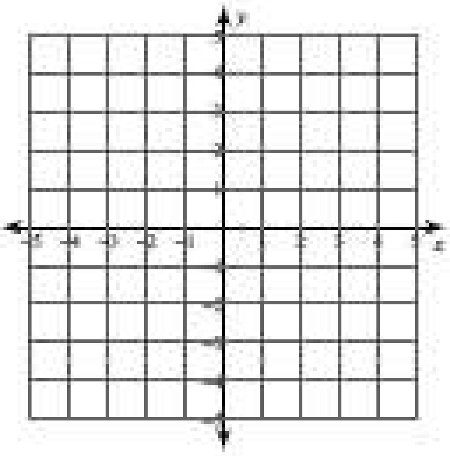
Solving Systems of Equations by Graphing

Date____________

Solve each system by graphing.

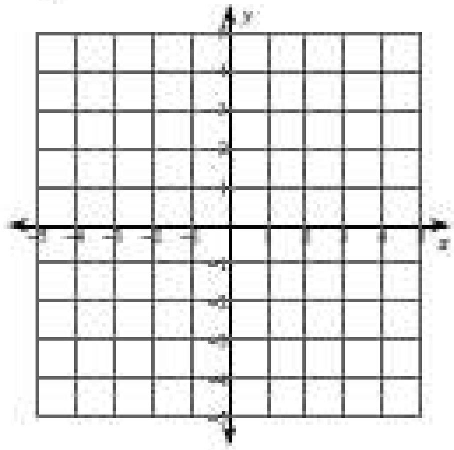
1) $y = -\frac{5}{3}x + 3$

$y = \frac{1}{3}x - 3$



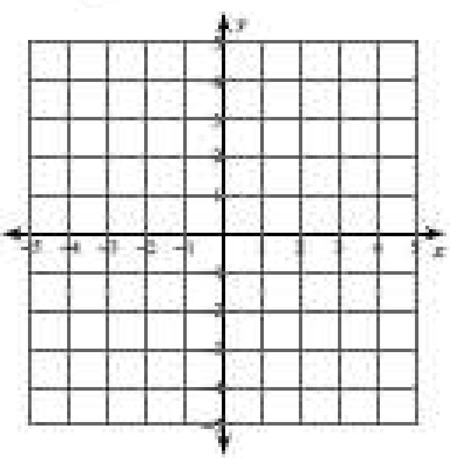
2) $y = 4x + 3$

$y = -x - 2$



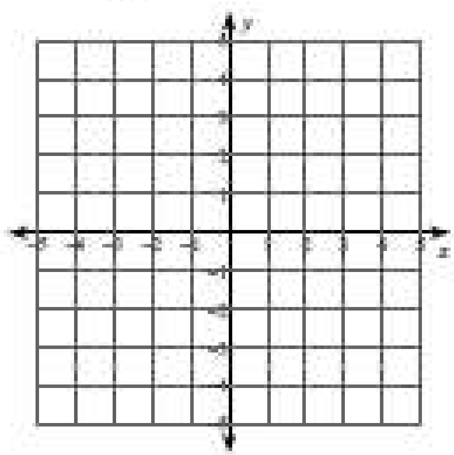
3) $y = -\frac{1}{2}x - 1$

$y = \frac{1}{4}x - 4$



4) $y = -1$

$y = -\frac{5}{2}x + 4$

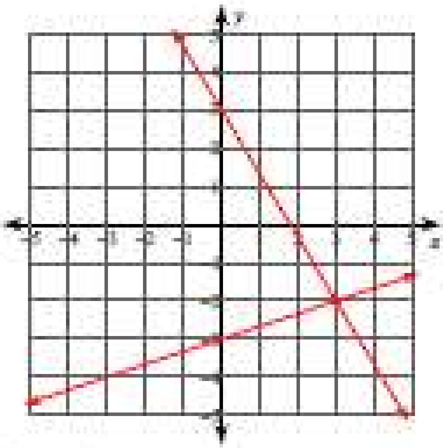


Solutions:

Solve each system by graphing.

1) $y = -\frac{5}{3}x + 3$

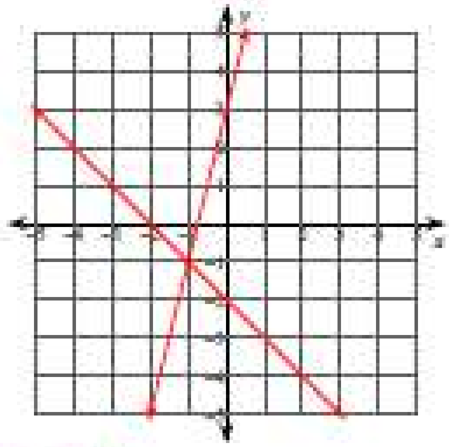
$y = \frac{1}{3}x - 3$



$(3, -2)$

2) $y = 4x + 3$

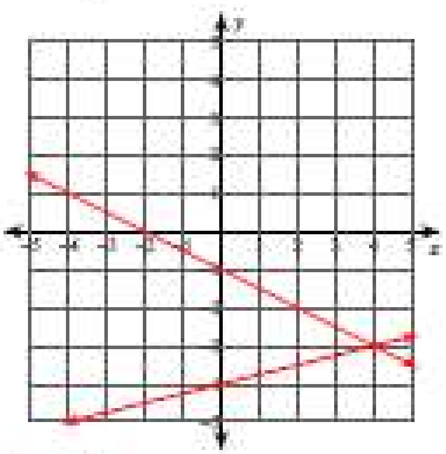
$y = -x - 2$



$(-1, -1)$

3) $y = -\frac{1}{2}x - 1$

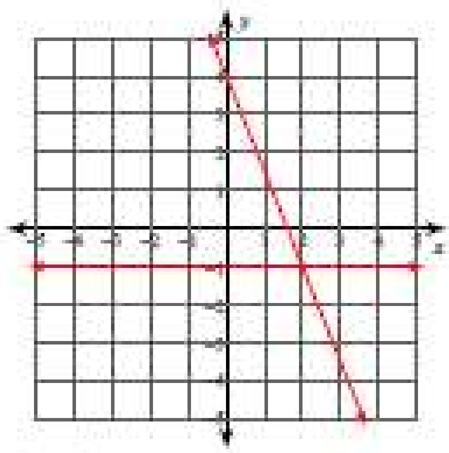
$y = \frac{1}{4}x - 4$



$(4, -3)$

4) $y = -1$

$y = -\frac{5}{2}x + 4$



$(2, -1)$

Simplifying Polynomials

Simplify the following polynomials:

1. $(5b + 6)(b + 2)$

2. $(6g^3 + 8g^2 - 11g + 3) - (7g^2 - 5g + 14)$

3. $(4x^2 - x - 12) + (5x^2 - 7x + 9)$

4. $(2h^2 + 5h - 1)(3h + 7)$

Why do we factor?

We factor to either…

1) To find the solutions of equations. To ask for the ____________________ is the same thing as asking for _____________, _____________ or __________________.

2) To simplify rational expression.

Factoring Polynomials:

Step 1: The first step in factoring is to factor out a Greatest Common Factor (GCF).

- Greatest Common Factor (GCF) is

  ____.

  - To factor out a GCF:
    1) Determine the GCF
    2) ______________ each term by the GCF
    3) Write the polynomial in factored form. NOTE: the GCF does not cancel out. It must be written outside the grouping symbols!

State the GCF of each polynomial.

Ex 1: $2x + 4$

Ex 2: $16x^3 - 24x^2$

Ex 3: $20x^4y^3 + 4xy^2 - 10x^5y^5z$

Ex 4: $3x^3 + 6x^2 - 2x - 4$

Factor out the GCF.

Ex 5: $3x^2 - 3x - 36$

Ex 6: $2x^2y - 50y$

Ex 7: $8x^3 - 16x + 64$

Ex 8: $12x^3 - 21x^2 - 28x$

Step 2:

Determine the number of terms in the polynomial

Why do care how many terms are in the polynomial?

- Binomial: ________ terms
- Trinomial: ________ terms
- "Quadnomial": ________ terms

Factoring "Quadnomials"

There are four terms in the polynomial, it is called a ______________________ and we use the ________________ method to factor.

To Factor by Grouping:

1) Factor out a _______ first. Be sure it is written outside the **parentheses** in the final answer.
2) Be sure the polynomial is in standard form, meaning…
   - Group the first two and second two terms together.
   - Factor out the __________ of each set. If it works, the remaining factor should be the same, and is, hence, a common factor.
   - Write the two factors.
   - Check by expanding, meaning distributing.

Ex 9: $x^3 + x^2 + 4x + 4$

Ex 10: $x^4 + x^2y^2 - 5x^2 - 5y^2$

Ex 11: $3w^3 - w^2 + 3w - 1$

Ex 12: $x^3 - 3x^2 - 16x + 48$

You Try:

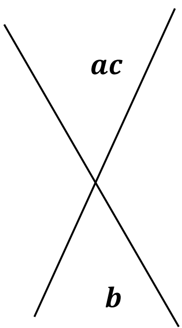
1.) $x^3 + 2x^2 + x + 2$	2.) $b^3 + b^2 - 2b - 2$	3.) $cb^3 - b^3 + 5c - 5$
4.) $x^3 + 2x^2 + 4x + 8$	5.) $2x^3 - 6x^2 + 3x - 9$	6.) $3x^3 + 6x^2 - 2x - 4$
7.) $4x^3 - 6x^2 + 10x - 15$	8.) $x^3 - 5x^2 - x + 5$	9.) $x^3 - 2x^2 - 11x + 12$
10.) $12x^3 - 9x^2 + 4x - 3$	11.) $2x^3 + 5x^2 + 6x + 15$	12.) $3x^3 - 4x^2 + 9x - 12$

Factoring Trinomials

To factor a trinomial in the form of $ax^2 + bx + c$

1) Be sure in standard form.
2) Identify $\boldsymbol{a}$, $\boldsymbol{b}$, & $\boldsymbol{c}$.
3) Multiply $\boldsymbol{ac}$ & write in top section of X
4) Place $\boldsymbol{b}$ in bottom section
5) Find two numbers that equal $\boldsymbol{ac}$ AND add to $\boldsymbol{b}$.
   i. Don't forget the signs matter.
   ii. Write in side sections.
6) Rewrite the trinomial as a "quadnomial". The numbers in the side sections split the middle term. Be sure you include the variables.
7) Factor by grouping
8) Write the polynomial in complete factor form.

$a =$ $b =$ $c =$



Team Practice:

1) $x^2 + 10x - 11$	2) $x^2 + 7x + 10$	3) $x^2 - 2x - 24$
4) $y^2 + 8y + 12$	5) $a^2 - 11a + 21$	6) $x^2 + 13x - 18$
7) $c^2 - 6c - 16$	8) $a^2 + 11ab - 30b^2$	9) $x^2 - 10x + 9$

10) $4x^2 + 8x + 3$	11) $3x^2 - 13x + 4$	12) $3x^2 + 10x + 8$

Difference of Squares

The expression $a^2 - b^2$ is the difference of two squares. There is a pattern to its factors.

$a^2 - b^2 = (a + b)(a - b)$ **OR** $a^2 - b^2 = (a - b)(a + b)$

Factor. Remember to look for a GCF first.

1) $4x^2 - 9$	2) $-49 + z^2$	3) $25x^2 - 9$
4) $3n^2 - 12$	5) $100 - 81y^2$	6) $-16m^2 + n^2$
7) $2n^2 - 98$	8) $n^2 + 4$	9) $4n^2 - 64$

Solving by Factoring

We factor to find the solutions of equations. To ask for the ____________ is the same thing as asking for _____________, ____________ or ________________.

The Solve by Factoring process requires four major steps:

1) Move all terms to one side of the equation, usually the left, using addition or subtraction.
2) Factor the equation completely.
3) Set each factor equal to zero and solve.
4) List each solution from **Step 3** as a solution to the original equation.

Solve by Factoring:

1) $(k+1)(k-5)=0$

2) $(2m+3)(4m+3)=0$

3) $x^2-11x+19=-5$

4) $n^2+7n+15=5$

5) $c^2-6c=16$

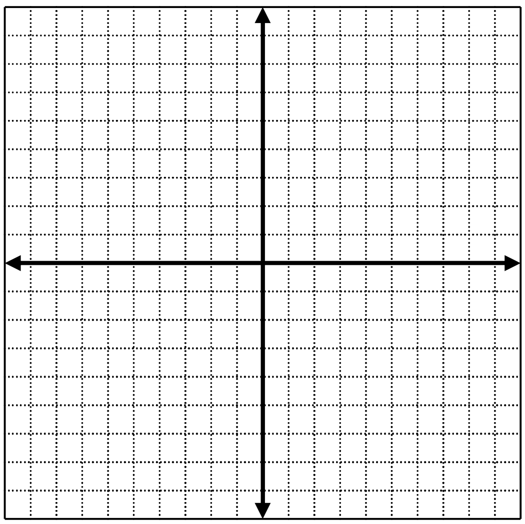
6) $3x^2=-10x-8$

<u>Solving Systems of Equations by Graphing</u>

Graph the following systems and write down the solution. Use two colors and a ruler.

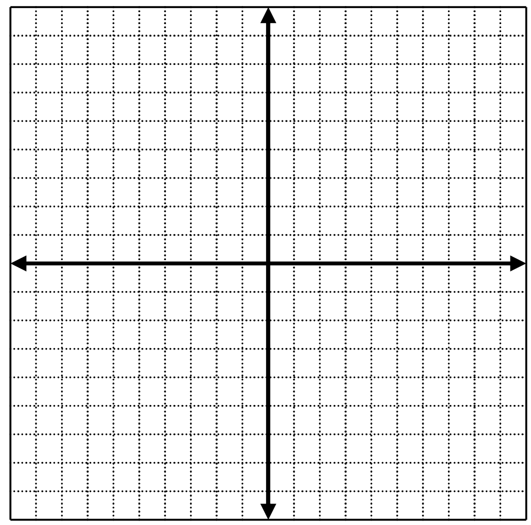
1) $\begin{cases} y = x^2 - 2x + 3 \\ y = x + 1 \end{cases}$

Solution(s): _______________



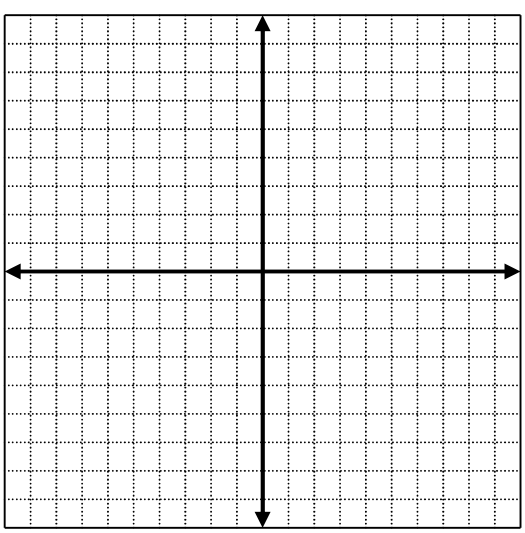
2) $\begin{cases} y = 4x + 5 \\ y = 1 \end{cases}$

Solution(s): _______________



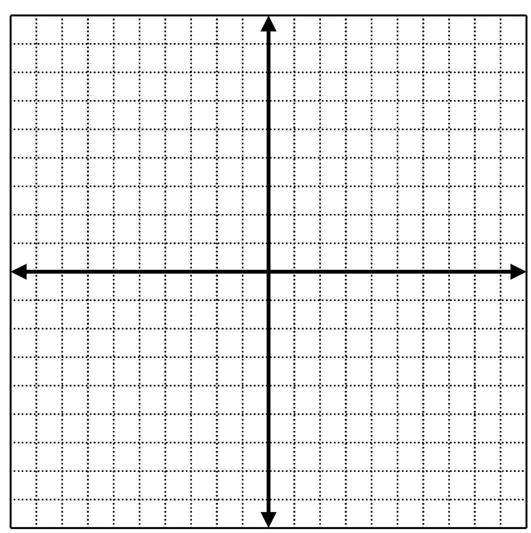
3) $\begin{cases} -2x + y = 6 \\ y = 2x + 8 \end{cases}$

Solution(s): _______________



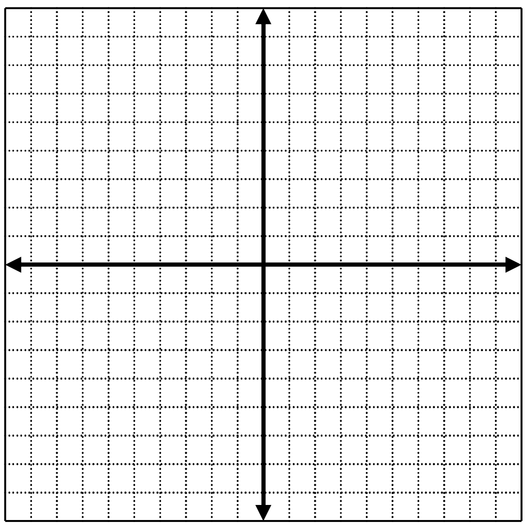
4) $\begin{cases} y = x^2 - 3x - 3 \\ y = -x + 5 \end{cases}$

Solution(s): _______________



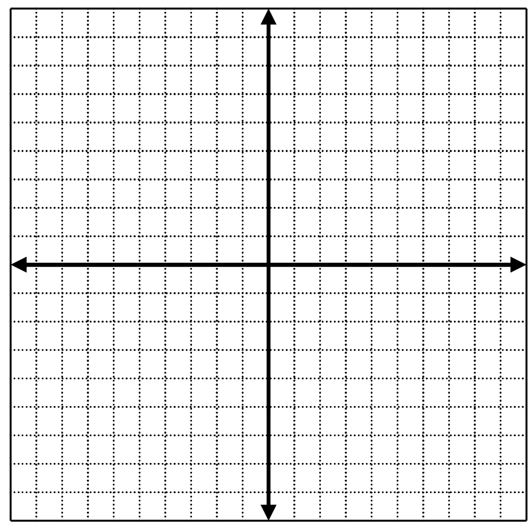
5) $\begin{cases} 3x + y = 12 \\ y = x \end{cases}$

Solution(s): ______________



6) $\begin{cases} y = 2x^2 \\ 2x - y = 4 \end{cases}$

Solution(s): ______________



Solutions:

Algebra 2
Unit 3 Quadratics Notes

Name: ____________________
Block: ________ Date: ____________

Simplifying Polynomials

Simplify the following polynomials:

1. $(5b + 6)(b + 2)$

$5b^2 + 10b + 6b + 12$

$5b^2 + 16b + 12$

2. $(6g^3 + 8g^2 - 11g + 3) + (-7g^2 + 5g - 14)$

$6g^3 + g^2 - 6g - 11$

3. $(4x^2 - x - 12) + (5x^2 - 7x + 9)$

$9x^2 - 8x - 3$

4. $(2h^2 + 5h - 1)(3h + 7)$

$6h^3 + 14h^2 + 15h^2 + 35h - 3h - 7$

$6h^3 + 29h^2 + 32h - 7$

Why do we factor?

We factor to either…

1) To find the solutions of equations. To ask for the solutions is the same thing as asking for roots, zero or x-intercepts

2) To simplify rational expression.

Factoring Polynomials:

Step 1: The first step in factoring is to factor out a greatest common factor.

- Greatest Common Factor (GCF) is a common factor of each term.
  ↳ greatest coefficient, greatest exponent
  - To factor out a GCF:
    1) Determine the GCF
    2) Factor each term by the GCF
    3) Write the polynomial in factored form. NOTE: the GCF does not cancel out. It must be written outside the grouping symbols!

State the GCF of each polynomial.

Ex 1: $2x + 4$

GCF: 2

Ex 2: $16x^3 - 24x^2$

GCF: $8x^2$

Ex 3: $20x^4y^3 + 4xy^2 - 10x^5y^5z$

GCF: $4xy^2$

Ex 4: $3x^3 + 6x^2 - 2x - 4$

GCF: none

Algebra 2
Unit 3 Quadratics Notes

Name: ______________________
Block: ________ Date: ______________

Factor out the GCF.

Ex 5: $3x^2 - 3x - 36$

$3(x^2 - x - 12)$

Ex 6: $2x^2y - 50y$

$2y(x^2 - 25)$

Ex 7: $8x^3 - 16x + 64$

$8(x^3 - 2x + 8)$

Ex 8: $12x^3 - 21x^2 - 28x$

$4x(3x^2 - 3x - 7)$

Step 2:
Determine the number of terms in the polynomial

Why do care how many terms are in the polynomial?

- Binomial: __2__ terms
- Trinomial: __3__ terms
- "Quadnomial": __4__ terms

Factoring "Quadnomials"

There are four terms in the polynomial, it is called a __"quadnomial"__ and we use the __grouping__ method to factor.

To Factor by Grouping:

1) Factor out a __GCF__ first. Be sure it is written outside the **parentheses** in the final answer.
2) Be sure the polynomial is in standard form, meaning…
   - Group the first two and second two terms together.
   - Factor out the __GCF__ of each set. If it works, the remaining factor should be the same, and is, hence, a common factor.
   - Write the two factors.
   - Check by expanding, meaning distributing.

Ex 9: $(x^3 + x^2)(+ 4x + 4)$

$x^2(x+1) + 4(x+1)$

$(x+1)(x^2+4)$

Ex 10: $(x^4 + x^2y^2)(- 5x^2 - 5y^2)$

$x^2(x^2+y^2) - 5(x^2+y^2)$

$(x^2+y^2)(x^2-5)$

Ex 11: $(3w^3 - w^2)(+ 3w - 1)$

$w^2(3w-1) + 1(3w-1)$

$(3w-1)(w^2+1)$

Ex 12: $(x^3 - 3x^2)(- 16x + 48)$

$x^2(x-3) - 16(x-3)$

$(x-3)(x^2-16)$*

*can be factored further… we will learn later in unit

Algebra 2
Unit 3 Quadratics Notes

Name: ______________________
Block: ________ Date: ______________

You Try:

1.) $(x^3 + 2x^2)(+ x + 2)$ $x^2(x+2)+1(x+2)$ $(x+2)(x^2+1)$	2.) $(b^3 + b^2)(- 2b - 2)$ $b^2(b+1)-2(b+1)$ $(b+1)(b^2-2)$	3.) $(cb^3 - b^3)(+ 5c - 5)$ $b^3(c-1)+5(c-1)$ $(c-1)(b^3+5)$
4.) $(x^3 + 2x^2)(+ 4x + 8)$ $x^2(x+2)+4(x+2)$ $(x+2)(x^2+4)$	5.) $(2x^3 - 6x^2)(+ 3x - 9)$ $2x^2(x-3)+3(x-3)$ $(x-3)(2x^2+3)$	6.) $(3x^3 + 6x^2)(- 2x - 4)$ $3x^2(x+2)-2(x+2)$ $(x+2)(3x^2-2)$
7.) $(4x^3 - 6x^2)(+ 10x - 15)$ $2x^2(2x-3)+5(2x-3)$ $(2x-3)(2x^2+5)$	8.) $(x^3 - 5x^2)(- x + 5)$ $x^2(x-5)-1(x-5)$ $(x-5)(x^2-1)$	9.) $(x^3 - 2x^2)(- 11x + 12)$ $x^2(x-2)-1(11x-12)$ prime ... cannot be factored
10.) $(12x^3 - 9x^2)(+ 4x - 3)$ $3x^2(4x-3)+1(4x-3)$ $(4x-3)(3x^2+1)$	11.) $(2x^3 + 5x^2)(+ 6x + 15)$ $x^2(2x+5)+3(2x+5)$ $(2x+5)(x^2+3)$	12.) $(3x^3 - 4x^2)(+ 9x - 12)$ $x^2(3x-4)+3(3x-4)$ $(3x-4)(x^2+3)$

Algebra 2
Unit 3 Quadratics Notes

Name: ____________________
Block: ________ Date: ____________

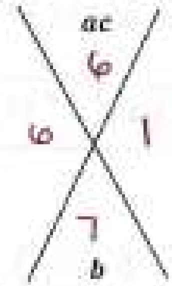
Factoring Trinomials

To factor a trinomial in the form of $ax^2 + bx + c$

1) Be sure in standard form.
2) Identify a, b, & c.
3) Multiply ac & write in top section of X
4) Place b in bottom section
5) Find two numbers that equal ac AND add to b.
   i. Don't forget the signs matter.
   ii. Write in side sections.
6) Rewrite the trinomial as a "quadnomial". The numbers in the side sections split the middle term. Be sure you include the variables.
7) Factor by grouping
8) Write the polynomial in complete factor form.

$3x^2+7x+2$

$a=3$ $b=7$ $c=2$

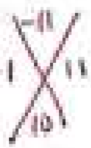


$(3x^2+6x)(+x+2)$
$3x(x+2)+1(x+2)$
$(x+2)(3x+1)$

Team Practice:

1) $x^2+10x-11$

$(x-1)(x+11)$

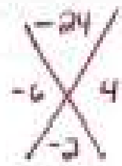


2) $x^2+7x+10$

$(x+5)(x+2)$

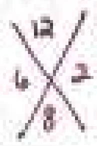
3) $x^2-2x-24$

$(x-6)(x+4)$



4) $y^2+8y+12$

$(y+6)(y+2)$



5) $a^2-11a+21$

prime

6) $x^2+13x-18$

prime

7) $c^2-6c-16$

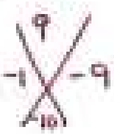
$(c-8)(c+2)$

8) $a^2+11ab-30b^2$

prime

9) $x^2-10x+9$

$(x-1)(x-9)$

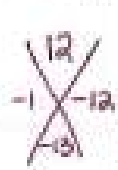


10) $4x^2+8x+3$

$(4x^2+6x)(+2x+3)$
$2x(2x+3)+1(2x+3)$
$(2x+3)(2x+1)$

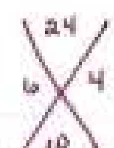
11) $3x^2-13x+4$

$(3x^2-x)(-12x+4)$
$x(3x-1)-4(3x-1)$
$(3x-1)(x-4)$



12) $3x^2+10x+8$

$(3x^2+6x)(+4x+8)$
$3x(x+2)+4(x+2)$
$(x+2)(3x+4)$



Algebra 2
Unit 3 Quadratics Notes

Name: ____________________
Block: ________ Date: ____________

Difference of Squares

The expression $a^2 - b^2$ is the difference of two squares. There is a pattern to its factors.

$$a^2 - b^2 = (a+b)(a-b) \quad \textbf{OR} \quad a^2 - b^2 = (a-b)(a+b)$$

Factor. Remember to look for a GCF first.

1) $4x^2 - 9$ $(2x-3)(2x+3)$	2) $-49 + z^2$ $z^2 - 49$ $(z-7)(z+7)$	3) $25x^2 - 9$ $(5x-3)(5x+3)$
4) $3n^2 - 12$ $3(n^2-4)$ $3(n-2)(n+2)$	5) $100 - 81y^2$ $(10-9y)(10+9y)$	6) $-16m^2 + n^2$ $-1(16m^2-n^2)$ $-1(4m-n)(4m+n)$
7) $2n^2 - 98$ $2(n^2-49)$ $2(n-7)(n+7)$	8) $n^2 + 4$ not difference prime... not a difference of perfect squares	9) $4n^2 - 64$ $4(n^2-16)$ $4(n-4)(n+4)$

Helpful Video Links & Apps at:

Khan Academy https://www.khanacademy.org/math/algebra

Math is power 4u videos http://www.mathispower4u.com/geometry.php

http://www.mathispower4u.com/algebra.php

Desmos Calculator https://www.desmos.com/testing/virginia/graphing